INDERSCIENCE PUBLISHERS
Linking academia, business and industry through research

International Journal of Operational Research

2018 Vol.31 No.2

Pages	Title and author(s)
141-163	**Single-vendor single-buyer optimal consignment policy with generic demand distribution by considering some realistic factors** S.M. Shahidul Islam; Mohammad Abdul Hoque **DOI**: 10.1504/IJOR.2018.089127
164-197	**Semi-infinite minimax fractional programming under (Φ, ρ)-V-invexity and generalised (Φ, ρ)-V-invexity. Optimality** Tadeusz Antczak **DOI**: 10.1504/IJOR.2018.089128
198-223	**Two techniques for solving nonlinear decreasing demand inventory system with shortage backorders** Ririn Diar Astanti; The Jin Ai; Hunyh Trung Luong; Hui-Ming Wee **DOI**: 10.1504/IJOR.2018.089129
224-244	**A methodological approach to assess the content of work in air cargo operations** Rossella Pozzi; Carlo Noè; Tommaso Rossi **DOI**: 10.1504/IJOR.2018.089130
245-261	**Supply chain system for time and quality dependent decaying items with multiple market demand and volume flexibility** Surbhi Singhal; S.R. Singh **DOI**: 10.1504/IJOR.2018.089131
262-280	**An inventory model for deteriorating items with constant, linear and quadratic holding cost - a comparative study in third order equation** C. Krishnamoorthi; V. Choudri **DOI**: 10.1504/IJOR.2018.089132

Log in Help Sitemap

INDERSCIENCE PUBLISHERS
Linking academia, business and industry through research

Home > International Journal of Operational Research

International Journal of Operational Research

This journal also publishes Open Access articles

Editor in Chief
Prof. Angappa Gunasekaran

ISSN online
1745-7653

ISSN print
1745-7645

12 issues per year
Subscription price

Scopus

Globalisation of market and operations places tremendous pressure in making timely and accurate decisions using data analysis and more accurate information. This signifies the importance of developing suitable operations research (OR) techniques and models. *IJOR* covers new theory and application of such techniques and models that include inventory, queuing, transportation, game theory, scheduling, project management, mathematical programming, decision-support systems, multi-criteria decision making, artificial intelligence, neural network, fuzzy logic, expert systems, simulation. New theories and applications of OR models are welcome.

About this journal | Editorial board | Submitting articles

Sign up for new issue alerts
Subscribe/buy articles/issues
View sample issue
Latest issue contents as RSS feed
Forthcoming articles
Journal information in easy print format (PDF)

Publishing with Inderscience: ethical guidelines (PDF)
View all calls for papers
Recommend to a librarian (PDF)
Feedback to Editor

Find related journals

Keep up-to-date

- Our Blog
- Follow us on Twitter
- Visit us on Facebook
- Our Newsletter (subscribe for free)
- RSS Feeds
- New issue alerts

Return to top

Two techniques for solving nonlinear decreasing demand inventory system with shortage backorders

Ririn Diar Astanti* and The Jin Ai

Department of Industrial Engineering,
Universitas Atma Jaya Yogyakarta,
Jl. Babarsari 43, Yogyakarta 55281, Indonesia
Email: ririn@staff.uajy.ac.id
Email: jinai@mail.uajy.ac.id
*Corresponding author

Hunyh Trung Luong

Industrial and Manufacturing Engineering,
School of Engineering and Technology,
Asian Institute of Technology,
P.O. Box 4, Klong Luang, Pathumthani 12120, Thailand
Email: luong@ait.ac.th

Hui-Ming Wee

Department of Industrial and Systems Engineering,
Chung Yuan Christian University,
200 Chung Pei Road, Chung Li District,
Taoyuan City, 32023, Taiwan
Email: weehm@cycu.edu.tw

Abstract: This paper considers an inventory model with nonlinear decreasing demand and shortage backorders. Two techniques are proposed to solve the problem. The first heuristic technique is based on cost reduction concept and the second one applies the particle swarm optimisation algorithm. The results from the two proposed techniques for variable replenishment interval policies are compared with those of fixed replenishment interval policy. The computational experiments show that the total cost resulted from variable replenishment interval policy is smaller than the fixed replenishment interval policy, especially when the demand rate is highly nonlinear.

Keywords: inventory policy; nonlinear decreasing demand; particle swarm optimisation; PSO; shortage backorders; fixed replenishment; variable replenishment.

Reference to this paper should be made as follows: Astanti, R.D., Ai, T.J., Luong, H.T and Wee, H-M. (2018) 'Two techniques for solving nonlinear decreasing demand inventory system with shortage backorders', *Int. J. Operational Research*, Vol. 31, No. 2, pp.198–223.

Biographical notes: Ririn Diar Astanti is an Assistant Professor in the Department of Industrial Engineering, Universitas Atma Jaya Yogyakarta, Indonesia. She received her BE in Industrial Engineering and MS in Information Technology Management from Institut Teknologi Sepuluh Nopember Surabaya, Indonesia, respectively. She received her Doctor of Engineering in Industrial Engineering and Management, Asian Institute of Technology, Thailand. Her research interests are in the field of inventory management and decision-making.

The Jin Ai is an Associate Professor in the Department of Industrial Engineering, Universitas Atma Jaya Yogyakarta, Indonesia. He received his Doctor of Engineering in Industrial Engineering and Management from Asian Institute of Technology, Thailand. His research interests include operation research and meta-heuristic.

Hunyh Trung Luong is an Associate Professor in Industrial and Manufacturing Engineering, School of Engineering and Technology, Asian Institute of Technology, Thailand. He received his Doctor of Engineering from Industrial Systems Engineering, Asian Institute of Technology, Thailand. His research interests include establishment of emergency inventory policies, inventory policies for perishable products, supply chain design, measures of bullwhip effect in supply chains, availability-based and reliability-based maintenance.

Hui-Ming Wee is a Distinguished Professor in Department of Industrial and Systems Engineering at Chung Yuan Christian University in Taiwan. He received his BS (honours) in Electrical and Electronic Engineering from Strathclyde University, UK, MEng in Industrial Engineering and Management from Asian Institute of Technology, Thailand, and PhD in Industrial Engineering from Cleveland State University, Ohio, USA. His research interests are in the field of production/inventory control, optimisation, and supply chain management. He has published more than 350 papers in refereed journals, international conferences, and book chapters. He has also co-edited four books and serves as editor/editorial board member for a number of international journals.

1 Introduction

Several inventory policy models in the past dealt with decreasing demand and shortage. The first inventory policy model with linear decreasing demand was proposed by Zhao et al. (2001). They developed a heuristic approach for variable replenishment interval policy. Later, Goyal and Giri (2003) developed a heuristic approach considering variable replenishment interval. Yang et al. (2004) developed the eclectic approach for solving inventory policy problem with nonlinear decreasing demand and variable replenishment interval policy. Wee (1995) developed an exact solution method for deteriorating fixed replenishment interval inventory problem considering exponentially decreasing demand pattern and shortage. At about the same time, Benkherouf (1995) developed an optimal procedure for decreasing demand, variable replenishment interval and finite planning horizon. Later, Zhou et al. (2004) developed an inventory model where shortage were classified into inventory followed by shortage (IFS) model and shortage followed by inventory (SFI) model. The difference between the two models is whether or not shortage is allowed in the last cycle.

With the rapid development of technological innovation, nowadays, the product life cycle of fashion product such as electronic devices and apparel products become shorter. In this study, we consider decreasing demand in the inventory policy problem since decreasing demand pattern exists in most industries, especially during the last phase of any product life cycle. Khouja (2005) formulated two-joint inventory models considering constant and linear decreasing demand. Rau and Ouyang (2007) proposed an algorithm for solving inventory models with linear increasing and decreasing demand. Yang et al. (2008) developed a collaborative vendor-buyer inventory model with exponentially decreasing demand and fixed replenishment interval. Rau and Ouyang (2008) proposed an integrated production-inventory policy considering linear increasing and decreasing demand. Omar (2009) considered a joint vendor and buyer lot sizing policy where the demand is linearly decreasing. Hsu et al. (2009) considered ordering policy model with triangle-shaped demand, where the demand is increasing during the introduction season and decreasing after the peak season. Skouri and Konstantaras (2009) proposed an order level inventory model with ramp type demand rate, in which ended by a period of decreasing demand rate, and partial backlog shortage. Panda (2011) addressed a joint lot-size and price inventory model with cost decrease under time and price dependent decreasing demand. Khanra and Chaudhuri (2011) discussed an order level inventory model with continuous quadratic function of time demand, constant deterioration rate, considering inflation, time value of money, and completely backlogged shortages. Yang et al. (2013) developed a model for pricing and replenishment strategy in a multi-market deteriorating product where the demand is exponentially decreasing with time and linearly decreasing with price. Lin et al. (2013) discussed a production inventory model where the production rate is dependent on demand rate and inventory level; the demand is exponentially decreasing and shortage is fully backordered. Sanni and Chukwu (2013) developed an inventory model for items with three-parameter Weibull distribution deterioration, ramp-type demand, and shortage backorder. Roy et al. (2013) developed an inventory replenishment policy model with general time-varying demand and shortages. Bera et al. (2013) developed an inventory model for a single deteriorating item with two separate storage warehouse, time and demand dependent selling price. Sicilia et al. (2014) analysed an inventory system with power demand, shortage backorder, and demand dependent production rate. Taleizadeh and Nematollahi (2014) proposed an inventory control problem for perishable item considering the time value of money, inflation, delay payment, and backordering. Krishnamoorthi and Panayappan (2014) investigated an inventory control policy for a single product during its product life cycle considering defective items and shortage backorder. Tyagi et al. (2014) proposed an inventory model for deteriorating item with stock-dependent demand, variable holding cost, non-instantaneous deteriorating, and partially backlogged shortages.

Our study considers nonlinear decreasing demand inventory system with variable replenishment interval, and develops two new heuristic techniques based on cost reduction concept and particle swarm optimisation (PSO). The remaining parts of this paper are organised as follows: Section 2 presents the mathematical modelling. In Section 3, the proposed heuristic technique based on cost reduction concept is developed, PSO is shown in Section 4. Section 5 explains the exact solution technique for fixed replenishment interval policy. Numerical experiments to illustrate the applicability of the proposed heuristic techniques and the exact solution technique are presented in Section 6. Comparisons among techniques are given in Section 7 and concluding remarks are given in Sections 8.

2 Mathematical modelling

The following assumptions based on the model developed by Wee (1995) are used for model development:

1 demand is known and decreases exponentially

2 the replenishment is made at time t_i ($i = 1, 2, \ldots, n$) where $t_1 = 0$

3 replenishment is instantaneous

4 the quantity received at t_i is used partly to meet accumulated shortages in the previous cycle from time s_{i-1} to t_i ($s_{i-1} < t_i$); ($i = 1, 2, \ldots, n$)

5 no shortages at the beginning ($t_1 = 0$) and the end of the planning horizon ($s_n = H$).

The notations used in this paper are defined below:

H length of the planning horizon under consideration

A initial demand rate

α parameter of the decreasing rate of demand rate

$f(t)$ instantaneous demand rate at time t, $f(t) = A \cdot e^{-\alpha t}$ $t \geq 0$

$I(t)$ inventory level at time $t \in [t_i, t_{i+1}]$ which is evaluated after the replenishment arrives at time t_i and before the replenishment arrives at time t_{i+1} in the i^{th} cycle $[t_i, t_{i+1}]$

c_1 ordering cost per order

c_2 holding cost per unit per unit time

c_3 shortage cost per unit per unit time

n number of replenishments

t_i the i^{th} replenishment time ($i = 1, 2, \ldots, n$)

s_i the shortage starting point of cycle i, which is the time at which the inventory level reaches zero in the i^{th} cycle $[t_i, t_{i+1}]$; ($i = 1, 2, \ldots, n - 1$).

Graphical representation of behaviour of the inventory system is shown in Figure 1.

Total inventory cost is expressed as:

$$TC\left(n, \{s_i\}, \{t_i\}\right) = nc_1 + c_2 \sum_{i=1}^{n} I_i + c_3 \sum_{i=1}^{n-1} S_i \tag{1}$$

where

I_i cumulative holding inventory during cycle i ($i = 1, 2, \ldots, n$), and is determined as:

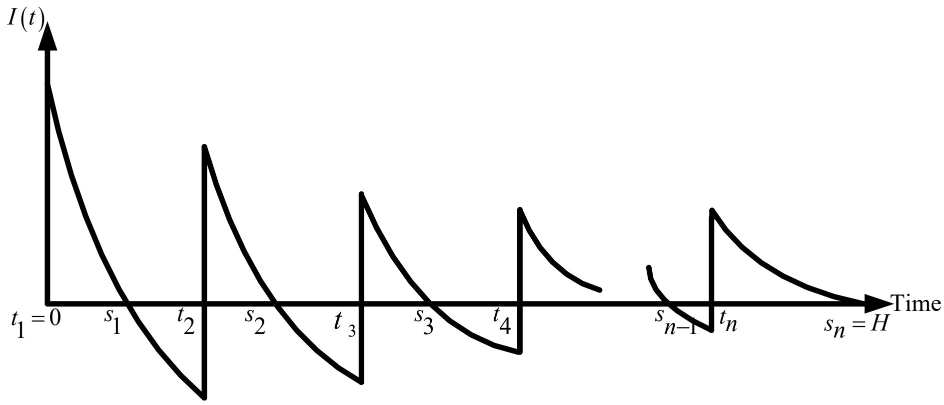
$$I_i = \int_{t_i}^{s_i} I(t)dt = \int_{t_i}^{s_i}\int_{t}^{s_i} f(\tau)d\tau dt \tag{2}$$

S_i cumulative shortage during cycle i ($i = 1, 2, \ldots, n - 1$), and is determined as:

$$S_i = \int_{s_i}^{t_{i+1}} \left(-I(t)\right)dt = \int_{s_i}^{t_{i+1}}\int_{s_i}^{t} f(\tau)d\tau dt \tag{3}$$

Two policies are considered in this paper. They are fixed replenishment interval and variable replenishment interval. In the first policy, the interval between two consecutive replenishment are exactly the same, i.e., $t_{i+1} - t_i = H / n$ for $i = 1, 2, \ldots, n - 1$ and $s_n - t_n = H / n$ (Wee, 1995). In the second policy, the intervals between two consecutive replenishments are varying. The solution procedures for the two policies are described in the following sections.

Figure 1 Graphical representation of the inventory system with decreasing demand and shortage backorders



3 Proposed heuristic technique based on cost reduction concept for variable replenishment interval policy

The proposed heuristic technique based on cost reduction concept seeks to determine the replenishment time $\{t_i\}$ and shortage point $\{s_i\}$. It consists of two steps. The first step is based on the consecutive methods developed by Wang (2002) to derive the replenishment time $\{t_i\}$. Wang's (2002) method is modified for exponential decreasing demand pattern. The second step is used to derive the shortage point $\{s_i\}$; it is based on the concept of cost reduction proposed by Astanti and Luong (2009). Detail explanation of each step is described in the following subsection.

3.1 Procedure to find $\{t_i\}$

The one time replenishment value Q is the quantity to fulfil demand for the whole cycle [0, H] (see Figure 2).

Figure 2 Graphical representation of a single period inventory system

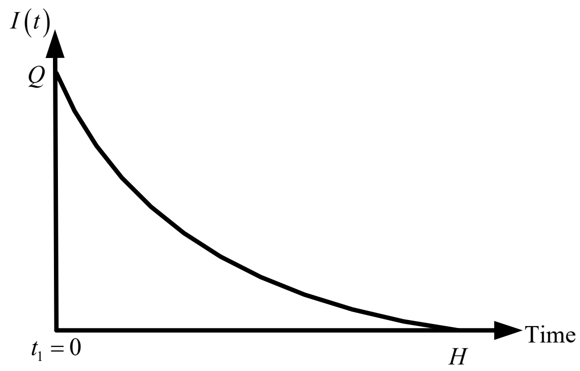
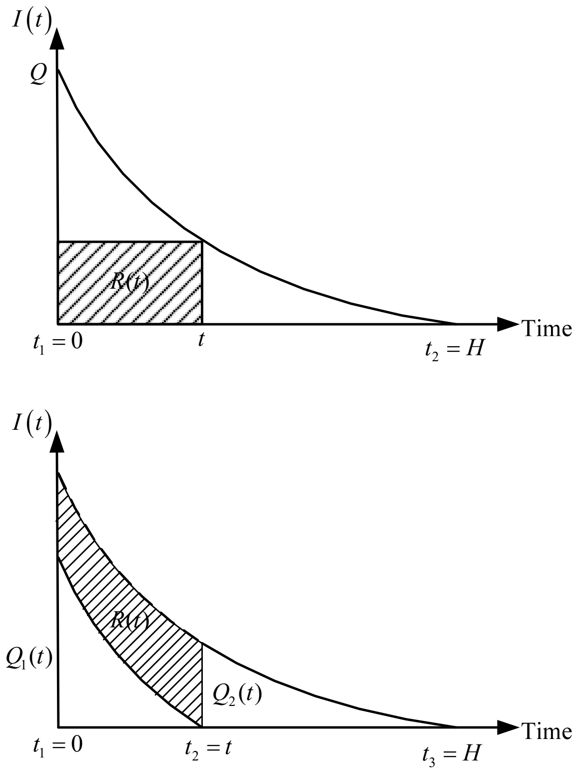


Figure 3 Reduction in holding cost with an additional replenishment



If two replenishments are placed at time arbitrary selected, the replenished quantity Q_1 at $t_1 = 0$ is used to fulfil the demand from [0, t_2], and the replenished quantity Q_2 at time t_2 is used to fulfil the demand from [t_2, H]. The reduction of holding cost by having an additional replenishment is investigated and the value is compared with the ordering cost. If the reduction of holding cost is greater than the ordering cost, then having an additional replenishment is implemented. This concept is illustrated in Figure 3.

The objective is to maximise cost reduction (*RC*) as follows:

$$\text{Maximise } RC_1^{(i)} = c_2\left(R(t)\right) - c_1 \tag{4}$$

Subject to

$$t_i \le t \le t_{i+1} \tag{5}$$

where

$RC_1^{(i)}$ cost reduction in cycle i

T(t) the reduction of holding inventory (as shown in Figure 3), and can be expressed as:

$$\begin{aligned} R(t) &= (t - t_i).Q_i(t) \\ &= (t - t_i).\int_t^{t_{i+1}} f(t)dt \\ &= (t - t_i)\int_t^{t_{i+1}} Ae^{-\alpha t}dt \\ &= -(t - t_i).\frac{A}{\alpha}.(e^{-\alpha.t_{i+1}} - e^{-\alpha.t}) \end{aligned} \tag{6}$$

By substituting equation (6) in to equation (5), the cost reduction $RC_1^{(i)}$ can be expressed as:

$$\text{Maximise } RC_1^{(i)} = c_2\left(-(t - t_i).\frac{A}{\alpha}.(e^{-\alpha.t_{i+1}} - e^{-\alpha.t})\right) - c_1 \tag{7}$$

Proposition 1: For each cycle i, there exists an optimal solution t^*, (where $t_i < t^* < t^{i+1}$), that maximise $RC_1^{(i)}$

Proof:

From the expression of $RC_1^{(i)}$ in (7), we have

$$\frac{dRC_1^{(i)}}{dt} = c_2\left\{-(t - t_i).Ae^{(-\alpha t)} - \frac{A\{e^{(-\alpha t_{i+1})} - e^{(-\alpha t)}\}}{\alpha}\right\}$$

$$\frac{d^2RC_1^{(i)}}{dt^2} = c_2 Ae^{(-\alpha t)}\left\{-2 + \alpha(t - t_i)\right\}$$

It is noted that

$$\frac{dRC_1^{(i)}}{dt}\Big|_{t=t_i} = -c_2 \frac{A\left[e^{-\alpha t_{i+1}} - e^{-\alpha t_i}\right]}{\alpha} > 0$$

and

$$\frac{dRC_1^{(i)}}{dt}\Big|_{t=t_i+1} = -c_2\left(t_{i+1} - t_i\right)Ae^{-\alpha t_{i+1}} < 0$$

Considering the following two situations:

1 If $t_i + \dfrac{2}{\alpha} \geq t_{i+1}$ then $\dfrac{d^2RC_1^{(i)}}{dt^2} < 0\ \forall t \in (t_i, t_{i+1})$. Hence, $RC_1^{(i)}$ is a convex function over (t_i, t_{i+1}) and the optimal solution t^* is the unique solution of the equation $\dfrac{dRC_1^{(i)}}{dt} = 0.$ This solution can be found by applying bisection method in the interval (t_i, t_{i+1}).

2 If $t_i + \dfrac{2}{\alpha} < t_{i+1}$ then $\dfrac{d^2RC_1^{(i)}}{dt^2} < 0\ \forall t \in \left(t_i, t_i + \dfrac{2}{\alpha}\right)$ and $\dfrac{d^2RC_1^{(i)}}{dt^2} > 0\ \forall t \in \left(t_i + \dfrac{2}{\alpha}, t_{i+1}\right)$.

   Hence, $\dfrac{dRC_1^{(i)}}{dt}$ is decreasing with respect to t in the interval $\left(t_i, t_i + \dfrac{2}{\alpha}\right)$ and increasing with respect to t in the interval $\left(t_i + \dfrac{2}{\alpha}, t_{i+1}\right)$.

   In this case, it can be seen that the optimal solution t^* exists and it is a unique solution when $\dfrac{dRC_1^{(i)}}{dt} = 0$ in the interval (t_i, t_{i+1}). This solution can be found by applying bisection method in the interval (t_i, t_{i+1}). (Q.E.D)

The necessary condition for optimising equation (7) is $dRC_1^{(i)}/dt = 0$ and can be written as:

$$c_2\left\{-\left(t - t_i\right).Ae^{(-\alpha t)} - \frac{A\left\{e^{(-\alpha t_{i+1})} - e^{(-\alpha t)}\right\}}{\alpha}\right\} = 0 \tag{8}$$

Since the closed form of equation (8) cannot be found, a bisection algorithm is used to derive the solution when $g(t_i)$ * $g(t_{i+1}) < 0$. The function $g(.)$ is the left hand side of equation (8).

The sufficient condition for maximum $RC_1^{(i)}$ at $t = t^*$ is $d^2RC_1^{(i)}/dt^2 < 0.$

This condition can be written as

$$-2A.e^{(-\alpha t)} + \left(A\alpha e^{(-\alpha t)}.\left(t - t_i\right)\right) < 0$$

$$Ae^{(-\alpha t)}\left\{-2 + \alpha\left(t - t_i\right)\right\} < 0$$

Since $e^{(-\alpha t)} > 0$ and $A > 0$, therefore the sufficient optimality condition hold true if $-2 + \alpha(t - t_i) < 0$.

The whole procedure to derive $\{t_i\}$ is an iterative procedure where an additional replenishment is evaluated for each step as described above. The iterative step is repeatedly performed until the optimal solution is derived.

3.2 *Procedure to find s_i*

As the inventory model considered in this paper is IFS model (see Figure 1), the procedure to find s_i is not performed until the last cycle. The cost reduction is defined as the difference between the reduction in holding cost (when shortage is allowed) and the holding cost (when shortage cost is not allowed). It can be formulated as follows:

$$\text{Maximise } RC_2^{(i)} = c_2\left(RH^{(i)}\right) - c_3\left(RS^{(i)}\right) \tag{9}$$

Subject to

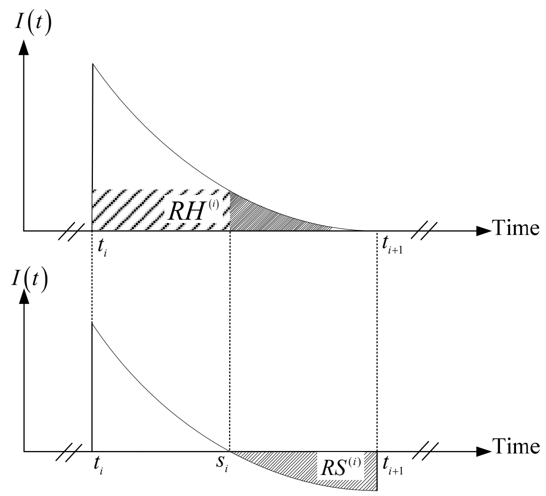
$$t_i \le s_i \le t_{i+1} \tag{10}$$

where

$RC_2^{(i)}$ cost reduction in cycle i

$RH^{(i)}$, $RS^{(i)}$ reduction in cumulative inventory and cumulative shortage in cycle i as shown in Figure 4.

Figure 4 Reduction in holding cost by incorporating shortage



Shortage is allowed if the maximum cost reduction is positive. Following the same procedure as Astanti and Luong (2009), the optimal value of s_i can be determined as:

$$s_i^* = \frac{c_2 t_i + c_3 t_{i+1}}{c_2 + c_3} \tag{11}$$

4 Proposed technique based on PSO for variable replenishment interval policy

PSO algorithm is a population-based search method that imitates physical movements of the individuals in the swarm as a searching method. It was inspired by social behaviour of bird flocking. Clerc (2006) stated that the basic principle of PSO is a set of moving particles that is placed in the search space. The PSO algorithm has been used for solving various optimisation problems such as warehouse layout design (Onut et al., 2008), job shop scheduling (Pongchairerks and Kachitvichyanukul, 2009), vehicle routing (Ai and Kachitvichyanukul, 2009) and joint buyer-vendor inventory problem (Taleizadeh et al., 2010). PSO is a population-based stochastic optimisation technique developed by Kennedy and Eberhart (1995). Particles have several choices to move in each period of time or iteration. The movement of particles in the basic in PSO formulation as follows:

$$\omega_{lh}(\tau+1) = w(\tau+1)\omega_{lh}(\tau) + c_p u\left(\psi_{lh} - \theta_{lh}(\tau)\right) + c_g u(\psi_{gh} - \theta_{lh}(\tau)) \tag{12}$$

$$\theta_{lh}(\tau+1) = \theta_{lh}(\tau) + \omega_{lh}(\tau+1) \tag{13}$$

where τ is iteration index, l is particle index, h is dimension index, u is uniform random number in interval [0, 1], $w(\tau)$ is inertia weight in the τ^{th} iteration, $\omega(\tau)$ is velocity of l^{th} particle at the h^{th} dimension in the τ^{th} iteration, $\theta(\tau)$ is position of l^{th} particle at the h^{th} dimension in the τ^{th} iteration, ψ_{lh} is personal best position (pbest) of l^{th} particle at the h^{th} dimension, ψ_{gh} is global best position (gbest) of l^{th} particle at the h^{th} dimension, c_p is personal best acceleration constant, and c_g is global best acceleration constant. Equation (13) shows that the particle position of next period is obtained from the sum of the current position with the velocity of the next period. Equation (12) showed that the velocity of next period obtained from the sum of the times of social or cognitive weights (w, c_p, and c_g) with current velocity, pbest, and gbest.

PSO algorithm for solving the problem in this paper will be explained in the following subsections.

4.1 Enumerative procedure to find n^* – Algorithm 1

The optimisation problem considered in this paper is to find number of replenishment (n), the set of shortage starting point of each cycle ($\{s_i\}$), and the set of replenishment time ($\{t_i\}$) in order to minimise the total cost (TC) in the equation (1). PSO algorithm presented in Algorithm 2 (Section 4.2) is able to derive the best value of set of shortage starting point of each cycle ($\{s_i^*\}$), and the set of replenishment time ($\{t_i^*\}$) given the fixed value of n. Therefore, it is still needed to find the optimal number of replenishment (n*). This PSO algorithm developed in this paper proposes an enumeration technique over PSO algorithm to handle this situation. The optimal value of n can be obtained by evaluating the $TC(n, \{t_i\}, \{s_i\})$ from equation (1) using enumeration technique starting from n = 1. The detail enumerative procedure for obtaining optimal value of n is presented in Algorithm 1.

Algorithm 1 Enumerative procedure to find n^*

1	Set $n = 1$. Evaluate $TC(n = 1)$ and set $TC^* = TC(n = 1)$
2	Set $n = n + 1$. Use PSO algorithm (Algorithm 2) to determine the best value of $\{t_i\}$ and $\{s_i\}$ and evaluate $TC(n, \{t_i\}, \{s_i\})$
3	If $TC(n, \{t_i\}, \{s_i\}) < TC^*$ then set $TC^* = TC(n, \{t_i\}, \{s_i\})$ and go to Step 2. Otherwise, set $n^* = n - 1$ and Stop.

4.2 PSO algorithm for the problem with fixed n – Algorithm 2

The details of the PSO algorithm for solving the problem presented in this paper are explained in Algorithm 2. This algorithm is developed based GLNPSO, a PSO with multiple social learning factors which are called local best, global best, and near neighbour best (Nguyen et al., 2010).

Algorithm 2 PSO algorithm for nonlinear decreasing demand inventory policies considering shortage backorders

1	Initialisation: Determine the number of particles, the particle's position and velocity.
2	Decode particles into solution which consists of $\{s_i\}$ and $\{t_i\}$ based on Algorithm 3.
3	Evaluate the particles, based on the objective function.
4	Update pbest value,
5	Update lbest, nbest, and gbest values,
6	Update velocity and position for each particle,
7	If the stopping criterion is reached, stop. Otherwise return to decoding step.

4.3 Particle representation of the problem with fixed n and decoding step – Algorithm 3

This subsection discusses how the particle can represent the problem. From the structure of the problem represented in Figure 1, it is clearly seen that $t_1 = 0$ and $s_n = H$. Therefore, for fixed value of n, the number of independent variable is $(n - 1)$, which are $t_2, t_3, \ldots, t_n$ and the boundary of the decision variable is $0 < t_2 < t_3 < \ldots < t_n < H$. Therefore, for fixed value of n, particle representation is random key of $(n - 1)$ elements, so the particle consists of $(n - 1)$ dimensions, and each dimensions position is limited from $0 < \theta_{li} < 1$. The decoding step from particle position into the set of shortage starting point of each cycle ($\{s_i\}$) and the set of replenishment time ($\{t_i\}$) is explained in Algorithm 3 as follows:

Algorithm 3 Decoding step

1	Sort the particle position from the smallest to the largest one
2	Calculate $t_i = \theta_{[i]} \cdot H$
3	Calculate $s_i = \dfrac{c_2 t_i + c_3 t_{i+1}}{c_2 + c_3}$

Example 1: Particle representation and decoding method.

Consider the problem with $c_2 = 10$, $c_3 = 40$, and $H = 4$. Given $n = 5$, therefore, the particle consists of $n - 1 = 4$ dimensions. Suppose there is a particle with position value of [0.8; 0.4; 0.1; 0.6]. Following algorithm 3, after sorting the position value we have $\theta_{[1]} = 0.1$, $\theta_{[2]} = 0.4$, $\theta_{[3]} = 0.6$, and $\theta_{[4]} = 0.8$.

Therefore, the value of t_i can be determined consequently:

$$t_2 = \theta_{[1]} \cdot H = 0.1 \cdot 4 = 0.4$$

$$t_3 = \theta_{[2]} \cdot H = 0.4 \cdot 4 = 1.6$$

$$t_4 = \theta_{[3]} \cdot H = 0.6 \cdot 4 = 2.4$$

$$t_5 = \theta_{[4]} \cdot H = 0.8 \cdot 4 = 3.2$$

It is noted that $t_1 = 0$ and $s_5 = H = 5$. Finally, the value of s_i can be determined accordingly:

$$s_1 = \frac{c_2 t_1 + c_3 t_2}{c_2 + c_3} = \frac{10 \cdot 0 + 40 \cdot 0.4}{10 + 40} = 0.32$$

$$s_2 = \frac{c_2 t_2 + c_3 t_3}{c_2 + c_3} = \frac{10 \cdot 0.4 + 40 \cdot 1.6}{10 + 40} = 1.36$$

$$s_3 = \frac{c_2 t_3 + c_3 t_4}{c_2 + c_3} = \frac{10 \cdot 1.6 + 40 \cdot 2.4}{10 + 40} = 2.24$$

$$s_4 = \frac{c_2 t_4 + c_3 t_5}{c_2 + c_3} = \frac{10 \cdot 2.4 + 40 \cdot 3.2}{10 + 40} = 3.04$$

Therefore, it can be concluded that particle with value of [0.8; 0.4; 0.1; 0.6] is corresponding with solution of the problem with $\{t_i\} = \{0.0; 0.4; 1.6; 2.4; 3.2\}$ and $\{s_i\} = \{0.32; 1.36; 2.24; 3.04; 4.00\}$.

5 Proposed technique for fixed replenishment interval

Referring to Wee (1995), the length of replenishment interval can be easily obtained as H / n, therefore, t_i can be formulated as

$$t_i = (i-1) \cdot \frac{H}{n} \text{ for } i = 2, 3, ..., n \tag{14}$$

Once the values of $\{t_i\}$ is known, the values of $\{s_i\}$ can be calculated using equation (11).

Since the service level (r) is defined as

$$s_i = r t_{i+1} + (1-r) t_i \text{ for } i = 2, 3, ..., n-1 \tag{15}$$

From equation (11) and equation (15), one can see that the value c_3 / (c_2 + c_3) represents the service level (r) as defined in Wee (1995).

The optimal value of n can be obtained by evaluating the $TC(n, \{t_i\}, \{s_i\})$ from equation (1) using enumeration technique starting at n = 1. The detail enumerative procedure for obtaining the optimal value of n is presented as follows:

Step 1 Set $n = 1$. Evaluate $TC(n = 1)$ and set $TC^* = TC(n = 1)$.

Step 2 Set $n = n + 1$. Calculate $\{t_i\}$ from equation (15) and $\{s_i\}$ from equation (11), and evaluate $TC(n, \{t_i\}, \{s_i\})$.

Step 3 If $TC(n, \{t_i\}, \{s_i\}) < TC^*$ then set $TC^* = TC(n, \{t_i\}, \{s_i\})$ and go to Step 2. Otherwise, set $n^* = n - 1$ and Stop.

6 Numerical example

In order to show the applicability of the proposed procedures presented above, the procedures are applied to solve an inventory policy problem with nonlinear decreasing demand and shortage backorders that is follow the example of Wee (1995). The problem description is as follow. The demand rate decreases exponentially following $f(t) = 500 \cdot e^{-0.98t}$ and the ordering cost (c_1) is \$250 per order, the carrying cost (c_2) is \$40 per unit per year, the backlogged shortage cost (c_2) is \$80 per unit per year and the system operates during a prescribed period of 4 year (H = 4).

The proposed procedures given in Sections 3 and 4 are applied to solve the problem example for variable replenishment interval policy. The results are presented in Tables 1 and 2. The fixed replenishment interval policy problem was solved using the procedure described in Section 5 and the results are presented in Table 3.

Table 1 Solution of the problem for variable replenishment interval policy based on cost reduction concept (TC^* = 4,645.7)

Cycle i	t_i	s_i
1	0.0000	0.1333
2	0.2000	0.3484
3	0.4226	0.5918
4	0.6763	0.8709
5	0.9681	1.2315
6	1.3632	1.6963
7	1.8629	2.0904
8	2.2041	2.4810
9	2.6195	2.9946
10	3.1821	4.0000

The applicability of the proposed procedures both for the variable replenishment policy and the fixed variable policy are shown by the results presented in Tables 1 to 3. Comparing the TC obtained from the problem example results, two proposed techniques based on reduction cost concept and PSO applied for IFS policy considering variable replenishment policy perform better than the exact solution technique of Wee (1995) applied for IFS policy for fixed replenishment interval policy. Furthermore, the proposed technique based on PSO perform better than the proposed technique based on reduction concept, since the first technique is able to provide smaller *TC* than the latter.

Table 2 Solution of the problem for variable replenishment interval policy based on PSO (TC^* = 4,543.80)

Cycle i	t_i	s_i
1	0.0000	0.1388
2	0.2082	0.3630
3	0.4404	0.6154
4	0.7030	0.9043
5	1.0050	1.2421
6	1.3607	1.6495
7	1.7940	2.1637
8	2.3486	2.8654
9	3.1237	4.0000

Table 3 Solution of the problem for solution of the problem for fixed interval policy (TC^* = 5,112.8)

Cycle i	t_i	s_i
1	0.0000	0.2667
2	0.4000	0.6667
3	0.8000	1.0667
4	1.2000	1.4667
5	1.6000	1.8667
6	2.0000	2.2667
7	2.4000	2.6667
8	2.8000	3.0667
9	3.2000	3.4667
10	3.6000	4.0000

7 Techniques comparison

7.1 Comparing the two proposed techniques for variable replenishment interval

In this section, the results from two proposed techniques for variable replenishment interval are compared. In order to compare two proposed techniques, many combinations of problem parameters are evaluated. The demand rate decreases exponentially following $f(t) = 500 \cdot e^{-\alpha t}$, where the value of α is set to be 0.02, 0.5, 0.98, and 2.0, respectively. These various demand rates are presented in Figure 5. It is noted that increasing the value of α is increasing the degree of nonlinearity on the demand rate. Various combinations of c_1, c_2, c_3, and H values are taken for comparing both techniques.

Table 4 Computational result in terms of n and TC on $\alpha = 0.02$

Problem parameter			*H = 1*					*H = 2*					*H = 4*				
			Variable (Reduction cost)		*Variable (PSO algorithm)*		Δ^1	*Variable (Reduction cost)*		*Variable (PSO algorithm)*		Δ^1	*Variable (Reduction cost)*		*Variable (PSO algorithm)*		Δ^1
c_1	c_2	c_3	n	TC^1	n	TC^2		n	TC^1	n	TC^2		n	TC^1	n	TC^2	
250	10	40	4	1,525.7	3	1,456.63	4.74%	8	3,010.3	6	2,851.07	5.58%	16	5,950.8	11	5,595.00	6.36%
250	10	80	4	1,566.8	3	1,510.87	3.70%	8	3,105.4	6	2,978.26	4.27%	16	6,150.6	12	5,871.24	4.76%
250	10	120	4	1,582.7	3	1,530.85	3.39%	8	3,142.0	6	3,026.16	3.83%	16	6,227.5	12	5,972.91	4.26%
250	20	40	4	1,927.7	4	1,899.73	1.47%	8	3,735.2	8	3,704.21	0.84%	16	7,302.1	15	7,243.14	0.81%
250	20	80	4	2,051.3	4	2,041.58	0.48%	8	4,020.6	8	4,009.66	0.27%	16	7,901.7	16	7,890.33	0.14%
250	20	120	4	2,104.2	4	2,099.36	0.23%	8	4,142.9	8	4,137.36	0.13%	16	8,158.6	16	8,153.23	0.07%
250	40	40	8	2,696.1	5	2,350.02	14.73%	16	5,301.8	9	4,556.37	16.36%	32	10,477.3	18	8,893.13	17.81%
250	40	80	8	2,876.4	6	2,663.98	7.97%	16	5,684.5	11	5,200.04	9.32%	32	11,252.6	20	10,218.3	10.12%
250	40	120	8	2,966.6	6	2,790.98	6.29%	16	5,875.8	11	5,484.65	7.13%	32	11,390.2	21	10,805.5	5.41%

Note: Proposed heuristic technique based on cost reduction concept vs. proposed technique based on PSO.

Table 5 Computational result in terms of n and TC on $\alpha = 0.5$

Problem parameter			*H = 1*					*H = 2*					*H = 4*				
			Variable (Reduction cost)		*Variable (PSO algorithm)*		Δ^1	*Variable (Reduction cost)*		*Variable (PSO algorithm)*		Δ^1	*Variable (Reduction cost)*		*Variable (PSO algorithm)*		Δ^1
c_1	c_2	c_3	n	TC^1	n	TC^2		n	TC^1	n	TC^2		n	TC^1	n	TC^2	
250	10	40	2	1,351.2	3	1,298.32	4.07%	4	2,277.9	5	2,259.51	0.81%	8	3,608.5	7	3,572.88	1.00%
250	10	80	2	1,390.6	3	1,338.29	3.91%	4	2,371.1	5	2,345.76	1.08%	8	3,747.7	7	3,734.68	0.35%
250	10	120	2	1,405.6	3	1,352.96	3.89%	4	2,406.5	5	2,377.91	1.20%	8	3,800.7	7	3,795.25	0.14%
250	20	40	4	1,725.7	4	1,705.25	1.20%	8	3,087.6	6	2,940.82	4.99%	10	4,812.2	9	4,673.73	2.96%
250	20	80	4	1,819.6	4	1,812.50	0.39%	8	3,260.5	6	3,177.60	2.61%	10	5,223.5	10	5,054.69	3.34%
250	20	120	4	1,859.5	4	1,855.94	0.19%	8	3,334.1	7	3,270.83	1.93%	10	5,397.5	10	5,211.85	3.56%
250	40	40	4	2,213.8	4	2,116.23	4.61%	8	3,737.5	8	3,649.81	2.40%	16	6,134.3	12	5,776.66	6.19%
250	40	80	4	2,451.4	5	2,360.69	3.84%	8	4,175.2	8	4,136.75	0.93%	16	6,737.3	13	6,582.94	2.34%
250	40	120	4	2,569.0	5	2,474.39	3.82%	8	4,391.9	9	4,352.05	0.92%	16	7,035.8	14	6,943.42	1.33%

Note: Proposed heuristic technique based on reduction cost concept vs. proposed technique based on PSO.

Table 6 Computational result in terms of n and TC on $\alpha = 0.98$

Problem parameter			*H = 1*					*H = 2*					*H = 4*				
			Variable (Reduction cost)		*Variable (PSO algorithm)*		Δ^1	*Variable (Reduction cost)*		*Variable (PSO algorithm)*		Δ^1	*Variable (Reduction cost)*		*Variable (PSO algorithm)*		Δ^1
c_1	c_2	c_3	n	TC^1	n	TC^2		n	TC^1	n	TC^2		n	TC^1	n	TC^2	
250	10	40	2	1,157.9	2	1,152.20	0.49%	4	1,815.9	4	1,808.61	0.40%	4	2,459.4	5	2,420.05	1.63%
250	10	80	2	1,186.5	2	1,184.85	0.14%	4	1,871.0	4	1,868.28	0.15%	4	2,545.3	5	2,505.45	1.59%
250	10	120	2	1,197.3	2	1,196.54	0.06%	4	1,891.9	4	1,890.23	0.09%	4	2,577.5	5	2,536.90	1.60%
250	20	40	4	1,573.4	3	1,510.77	4.15%	4	2,461.2	5	2,378.32	3.48%	8	3,338.6	6	3,220.08	3.68%
250	20	80	4	1,645.2	3	1,608.99	2.25%	4	2,631.8	5	2,546.26	3.36%	8	3,522.3	7	3,448.44	2.14%
250	20	120	4	1,675.6	3	1,647.77	1.69%	4	2,703.0	5	2,614.12	3.40%	8	3,599.1	7	3,539.21	1.69%
250	40	40	4	1,962.9	4	1,889.03	3.91%	8	3,143.1	6	2,970.87	5.80%	10	4,181.3	8	4,031.75	3.71%
250	40	80	4	2,146.7	4	2,115.90	1.46%	8	3,419.9	7	3,344.84	2.24%	10	4,645.7	9	4,543.80	2.24%
250	40	120	4	2,236.9	5	2,218.44	0.83%	8	3,555.7	7	3,509.19	1.33%	10	4,871.3	9	4,778.06	1.95%

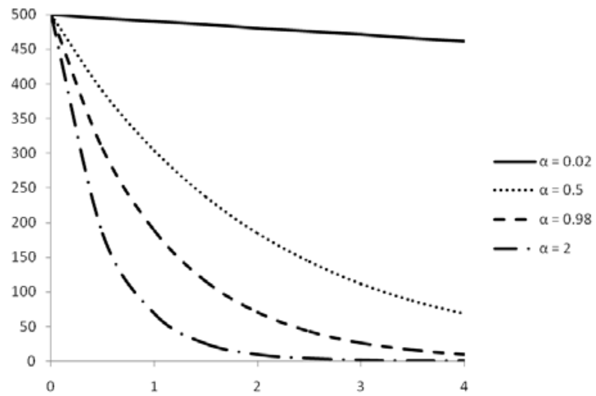
Note: Proposed heuristic technique based on reduction cost concept vs. proposed technique based on PSO.

Table 7 Computational result in terms of n and TC on $\alpha = 2$

Problem parameter			*H = 1*					*H = 2*					*H = 4*				
			Variable (Reduction cost)		*Variable (PSO algorithm)*		Δ^1	*Variable (Reduction cost)*		*Variable (PSO algorithm)*		Δ^1	*Variable (Reduction cost)*		*Variable (PSO algorithm)*		Δ^1
c_1	c_2	c_3	n	TC^1	n	TC^2		n	TC^1	n	TC^2		n	TC^1	n	TC^2	
250	10	40	2	893.0	2	890.55	0.27%	2	1,151.4	2	1,148.45	0.26%	2	1,237.6	2	1,234.69	0.24%
250	10	80	2	908.2	2	907.47	0.08%	2	1,172.8	2	1,172.03	0.07%	2	1,260.3	2	1,259.49	0.06%
250	10	120	2	913.8	2	913.50	0.03%	2	1,180.8	2	1,180.40	0.03%	2	1,268.7	2	1,268.29	0.03%
250	20	40	2	1,238.0	3	1,221.88	1.32%	4	1,643.0	3	1,575.57	4.28%	8	1,763.6	3	1,712.90	2.96%
250	20	80	2	1,286.0	3	1,277.08	0.70%	4	1,708.2	3	1,659.08	2.96%	4	1,834.2	3	1,802.59	1.75%
250	20	120	2	1,305.7	3	1,298.64	0.54%	4	1,735.1	3	1,691.34	2.59%	4	1,863.2	3	1,837.13	1.42%
250	40	40	4	1,608.9	3	1,531.99	5.02%	4	2,112.0	4	2,029.19	4.08%	4	2,337.7	4	2,239.55	4.38%
250	40	80	4	1,717.9	3	1,693.76	1.43%	4	2,286.0	4	2,247.68	1.70%	4	2,527.2	5	2,453.38	3.01%
250	40	120	4	1,770.5	4	1,759.63	0.62%	4	2,368.4	5	2,337.09	1.34%	4	2,616.5	5	2,548.60	2.66%

Note: Proposed heuristic technique based on reduction cost concept vs. PSO.

Figure 5 Various demand rates



The computational results of these problems are presented in Tables 4 to 7. Parameter Δ^1 is introduced here for comparing the objective function of proposed technique based on reduction cost concept over the objective function of proposed technique based on PSO. The Δ^1 is calculated using following equation

$$\Delta^1 = \frac{TC^1 - TC^2}{TC^2} \times 100\% \tag{16}$$

where

TC^1 the objective function of proposed technique based on reduction cost concept

TC^2 the objective function of proposed technique based on PSO.

Based on Tables 4 to 7, it can be concluded that proposed technique based on PSO perform better than that of based on reduction cost concept.

7.2 Comparing variable and fixed replenishment interval

Since the proposed technique based on PSO is perform better than that of based on reduction cost concept, the results from proposed technique based on PSO for variable interval replenishment policy is compared with the fixed interval replenishment policy in term of number of replenishment (n) and its corresponding TC. The results are then provided in Tables 8 to 11.

Parameter Δ^2 is introduced here for comparing the objective function of variable interval and fixed interval replenishment policies. The Δ^2 is calculated using following equation

$$\Delta^2 = \frac{TC^3 - TC^2}{TC^2} \times 100\% \tag{17}$$

where

TC^3 the objective function of fixed interval replenishment policy

TC^2 the objective function of variable interval replenishment policy.

Table 8 Comparison of fixed and variable interval replenishment policies for $\alpha = 0.02$

Problem parameter			*H = 1*					*H = 2*					*H = 4*				
			Fixed (*Wee, 1995*)		*Variable* (*PSO algorithm*)		Δ^2	*Fixed* (*Wee, 1995*)		*Variable* (*PSO algorithm*)		Δ^2	*Fixed* (*Wee, 1995*)		*Variable* (*PSO algorithm*)		Δ^2
c_1	c_2	c_3	n	TC^3	n	TC^2		n	TC^3	n	TC^2		n	TC^3	n	TC^2	
250	10	40	3	1,464.1	3	1,456.63	0.51%	6	2,859.6	6	2,851.07	0.30%	11	5,604.8	11	5,595.00	0.18%
250	10	80	3	1,513.0	3	1,510.87	0.14%	6	2,980.7	6	2,978.26	0.08%	12	5,873.6	12	5,871.24	0.04%
250	10	120	3	1,531.8	3	1,530.85	0.06%	6	3,027.2	6	3,026.16	0.03%	12	5,974.0	12	5,972.91	0.02%
250	20	40	4	1,927.0	4	1,899.73	1.44%	8	3,733.5	8	3,704.21	0.79%	15	7,275.5	15	7,243.14	0.45%
250	20	80	4	2,050.9	4	2,041.58	0.46%	8	4,019.6	8	4,009.66	0.25%	16	7,900.0	16	7,890.33	0.12%
250	20	120	4	2,103.9	4	2,099.36	0.22%	8	4,142.2	8	4,137.36	0.12%	16	8,157.5	16	8,153.23	0.05%
250	40	40	5	2,436.2	5	2,350.02	3.67%	10	4,652.8	9	4,556.37	2.12%	18	8,999.4	18	8,893.13	1.20%
250	40	80	6	2,690.7	6	2,663.98	1.00%	11	5,231.8	11	5,200.04	0.61%	21	10,242.2	20	10,218.30	0.23%
250	40	120	6	2,805.3	6	2,790.98	0.51%	11	5,502.1	11	5,484.65	0.32%	22	10,816.8	21	10,805.50	0.10%

Table 9 Comparison of fixed and variable interval replenishment policies for $\alpha = 0.5$

Problem parameter			$H = 1$					$H = 2$					$H = 4$				
			Fixed (*Wee, 1995*)		*Variable* (*PSO algorithm*)		Δ^2	*Fixed* (*Wee, 1995*)		*Variable* (*PSO algorithm*)		Δ^2	*Fixed* (*Wee, 1995*)		*Variable* (*PSO algorithm*)		Δ^2
c_1	c_2	c_3	n	TC^3	n	TC^2		n	TC^3	n	TC^2		n	TC^3	n	TC^2	
250	10	40	3	1,299.8	3	1,298.32	0.11%	5	2,270.5	5	2,259.51	0.49%	7	3,690.4	7	3,572.88	3.29%
250	10	80	3	1,339.0	3	1,338.29	0.05%	5	2,360.8	5	2,345.76	0.64%	8	3,867.8	7	3,734.68	3.56%
250	10	120	3	1,354.0	3	1,352.96	0.08%	5	2,395.2	5	2,377.91	0.73%	8	3,930.9	7	3,795.25	3.57%
250	20	40	4	1,715.1	4	1,705.25	0.58%	6	2,960.4	6	2,940.82	0.67%	10	4,815.4	9	4,673.73	3.03%
250	20	80	4	1,815.2	4	1,812.50	0.15%	6	3,198.3	6	3,177.60	0.65%	10	5,232.1	10	5,054.69	3.51%
250	20	120	4	1,857.7	4	1,855.94	0.10%	7	3,293.0	7	3,270.83	0.68%	11	5,398.8	10	5,211.85	3.59%
250	40	40	5	2,160.1	4	2,116.23	2.07%	8	3,697.2	8	3,649.81	1.30%	12	5,957.9	12	5,776.66	3.14%
250	40	80	5	2,375.0	5	2,360.69	0.61%	8	4,169.6	8	4,136.75	0.79%	14	6,804.0	13	6,582.94	3.36%
250	40	120	5	2,481.6	5	2,474.39	0.29%	9	4,383.0	9	4,352.05	0.71%	14	7,188.4	14	6,943.42	3.53%

Table 10 Comparison of fixed and variable interval replenishment policies in terms for $\alpha = 0.98$

Problem parameter			*H = 1*					*H = 2*					*H = 4*				
			Fixed (*Wee, 1995*)		*Variable* (*PSO algorithm*)			*Fixed* (*Wee, 1995*)		*Variable* (*PSO algorithm*)			*Fixed* (*Wee, 1995*)		*Variable* (*PSO algorithm*)		
c_1	c_2	c_3	n	TC^3	n	TC^2	Δ^2	n	TC^3	n	TC^2	Δ^2	n	TC^3	n	TC^2	Δ^2
250	10	40	2	1,152.8	2	1,152.20	0.05%	4	1,849.4	4	1,808.61	2.26%	5	2,720.4	5	2,420.05	12.41%
250	10	80	2	1,188.7	2	1,184.85	0.32%	4	1,920.6	4	1,868.28	2.80%	5	2,844.5	5	2,505.45	13.53%
250	10	120	2	1,202.2	2	1,196.54	0.47%	4	1,947.4	4	1,890.23	3.02%	5	2,890.7	5	2,536.90	13.95%
250	20	40	3	1,516.1	3	1,510.77	0.35%	5	2,429.5	5	2,378.32	2.15%	7	3,596.8	6	3,220.08	11.70%
250	20	80	3	1,613.6	3	1,608.99	0.29%	5	2,618.2	5	2,546.26	2.83%	8	3,901.6	7	3,448.44	13.14%
250	20	120	3	1,654.6	3	1,647.77	0.41%	5	2,697.0	5	2,614.12	3.17%	8	4,017.3	7	3,539.21	13.51%
250	40	40	4	1,918.0	4	1,889.03	1.53%	6	3,040.6	6	2,970.87	2.35%	9	4,484.9	8	4,031.75	11.24%
250	40	80	4	2,127.1	4	2,115.90	0.53%	7	3,431.1	7	3,344.84	2.58%	10	5,112.8	9	4,543.80	12.52%
250	40	120	5	2,228.1	5	2,218.44	0.44%	7	3,612.0	7	3,509.19	2.93%	11	5,398.3	9	4,778.06	12.98%

Table 11 Comparison of fixed and variable interval replenishment policies for $\alpha = 2$

Problem parameter			*H = 1*					*H = 2*					*H = 4*				
			Fixed (Wee, 1995)		*Variable (PSO algorithm)*		Δ^2	*Fixed (Wee, 1995)*		*Variable (PSO algorithm)*		Δ^2	*Fixed (Wee, 1995)*		*Variable (PSO algorithm)*		Δ^2
c_1	c_2	c_3	n	TC^3	n	TC^2		n	TC^3	n	TC^2		n	TC^3	n	TC^2	
250	10	40	2	902.6	2	890.55	1.35%	2	1,265.3	2	1,148.45	10.17%	2	1,607.6	2	1,234.69	30.20%
250	10	80	2	925.3	2	907.47	1.96%	2	1,302.5	2	1,172.03	11.13%	2	1,632.6	2	1,259.49	29.62%
250	10	120	2	933.7	2	913.50	2.21%	2	1,315.6	2	1,180.40	11.45%	2	1,640.7	2	1,268.29	29.36%
250	20	40	2	1,231.8	3	1,221.88	0.81%	3	1,745.2	3	1,575.57	10.77%	4	2,387.7	3	1,712.90	39.40%
250	20	80	3	1,300.8	3	1,277.08	1.86%	3	1,870.8	3	1,659.08	12.76%	4	2,537.6	3	1,802.59	40.78%
250	20	120	3	1,327.4	3	1,298.64	2.21%	3	1,920.1	3	1,691.34	13.53%	4	2,593.9	3	1,837.13	41.19%
250	40	40	3	1,548.2	3	1,531.99	1.06%	4	2,229.4	4	2,029.19	9.87%	6	3,108.0	4	2,239.55	38.78%
250	40	80	3	1,721.8	3	1,693.76	1.66%	5	2,497.2	4	2,247.68	11.10%	7	3,493.2	5	2,453.38	42.38%
250	40	120	4	1,796.0	4	1,759.63	2.07%	5	2,619.3	5	2,337.09	12.08%	7	3,652.7	5	2,548.60	43.32%

Based on Tables 8 to 11, it can be seen that proposed technique based on PSO for variable interval replenishment policy perform better than that of Wee's (1995) method for fixed interval replenishment policy.

8 Conclusions

In this study, two techniques are proposed to solve the nonlinear decreasing demand inventory system with shortage backorders. The first heuristic technique is based on cost reduction concept and the second one applies the PSO algorithm. The results from the two proposed techniques for variable replenishment interval policies are compared with the fixed replenishment interval policy. Based on the numerical example in this paper, it can be seen that the proposed heuristic technique based on PSO for variable replenishment policy result in a lower total inventory cost, especially when the demand rate is highly nonlinear. Therefore, for highly nonlinear decreasing demand, variable replenishment policy should be recommended.

Acknowledgements

The authors would like to thank the editor and the anonymous referees for their valuable comments and suggestions to improve the presentation of this paper. The first author received the Scheme for Academic Mobility and Exchange (SAME) Grant FY 2012 from the Directorate of Higher Education, Ministry of Education, Republic of Indonesia while initiating this research work. The second and the first authors also received International Research Collaboration and Scientific Publication Grant No. 086/SP2H/PL/DIT.LITABMAS/V/2013, No. 1317/K5/KM/2014, and No. 005/HB-LIT/III/2015 from the Directorate of Higher Education, Ministry of Education, Republic of Indonesia while completing this research work.

References

Ai, T.J. and Kachitvichyanukul, V. (2009) 'A particle swarm optimization for the vehicle routing problem with simultaneous pickup and delivery', *Computers & Operations Research*, Vol. 36, No. 5, pp.1693–1702, DOI: 10.1016/j.cor.2008.04.003.

Astanti, R.D. and Luong, H.T. (2009) 'A heuristic technique for inventory replenishment policy with increasing demand pattern and shortage allowance', *The International Journal of Advanced Manufacturing Technology*, Vol. 41, Nos. 11–12, pp.1199–1207, DOI: 10.1007/s00170-008-1566-6.

Benkherouf, L. (1995) 'On an inventory model with deteriorating items and decreasing time-varying demand and shortages', *European Journal of Operational Research*, Vol. 86, No. 2, pp.293–299, DOI: 10.1016/0377-2217(94)00101-H.

Bera, U.K., Bhunia, A.K. and Maiti, M. (2013) 'Optimal partial backordering two-storage inventory model for deteriorating items with variable demand', *International Journal of Operational Research*, Vol. 16, No. 1, pp.96–112, DOI: 10.1504/IJOR.2013.050542.

Clerc, M. (2006) *Particle Swarm Optimization*, ISTE, London.

Goyal, S.K. and Giri, B.C. (2003) 'A simple rule for determining replenishment intervals of an inventory item with linear decreasing demand rate', *International Journal of Production Economics*, Vol. 83, No. 2, pp.139–142, DOI: 10.1016/S0925-5273(02)00303-1.

Hsu, P.H., Teng, H.M. and Wee, H.M. (2009) 'Optimal lot sizing for deteriorating items with triangle-shaped demand and uncertain lead time', *European Journal of Industrial Engineering*, Vol. 3, No. 3, pp.247–260, DOI: 10.1504/EJIE.2009.025047.

Kennedy, J. and Eberhart, R. (1995) 'Particle swarm optimization', *Proceedings of IEEE International Conference on Neural Networks*, Vol. 4, pp.1942–1948.

Khanra, S. and Chaudhuri, K. (2011) 'On an EOQ model with SFI policy for a time quadratic demand', *International Journal of Operational Research*, Vol. 10, No. 3, pp.257–276, DOI: 10.1504/IJOR.2011.038901.

Khouja, M. (2005) 'Joint inventory and technology selection decisions', *Omega*, Vol. 33, No. 1, pp.47–53, DOI: 10.1016/j.omega.2004.03.006.

Krishnamoorthi, C. and Panayappan, P. (2014) 'An inventory model for product life cycle (growth stage) with defective items and shortages', *International Journal of Operational Research*, Vol. 19, No. 1, pp.1–20, DOI: 10.1504/IJOR.2014.057841.

Lin, J., Chao, H.C.J. and Julian, P. (2013) 'Planning horizon for production inventory models with production rate dependent on demand and inventory level', *Journal of Applied Mathematics*, No. 1, pp.1–9, Article ID 961258, doi: 10.1155/2013/961258.

Nguyen, S., Ai, T.J. and Kachitvichyanukul, V. (2010) *Object Library for Evolutionary Techniques ET-Lib: User's Guide*, High Performance Computing Group, Asian Institute of Technology, Thailand.

Omar, M. (2009) 'An integrated equal-lots policy for shipping a vendor's final production batch to a single buyer under linearly decreasing demand', *International Journal of Production Economics*, Vol. 118, No. 1, pp.185–188, DOI: 10.1016/j.ijpe.2008.08.020.

Onut, S., Tuzkaya, U.R. and Dogac, B. (2008) 'A particle swarm optimization algorithm for the multiple-level warehouse layout design problem', *Computers & Industrial Engineering*, Vol. 54, No. 4, pp.783–799, DOI: 10.1016/j.cie.2007.10.012.

Panda, S. (2011) 'Optimal pricing and replenishment policy in a declining price sensitive environment under continuous unit cost decrease', *International Journal of Mathematics in Operational Research*, Vol. 3, No. 4, pp.431–450, DOI: 10.1504/IJMOR.2011.040877.

Pongchairerks, P. and Kachitvichyanukul, V. (2009) 'A two-level particle swarm optimisation algorithm on job-shop scheduling problems', *International Journal of Operational Research*, Vol. 4, No. 4, pp.390–411, DOI: 10.1504/IJOR.2009.023535.

Rau, H. and Ouyang, B.C. (2007) 'A general and optimal approach for three inventory models with a linear trend in demand', *Computers & Industrial Engineering*, Vol. 52, No. 4, pp.521–532, DOI: 10.1016/j.cie.2007.03.001.

Rau, H. and Ouyang, B.C. (2008) 'An optimal batch size for integrated production-inventory policy in a supply chain', *European Journal of Operational Research*, Vol. 185, No. 2, pp.619–634, DOI: 10.1016/j.ejor.2007.01.017.

Roy, T.S., Ghosh, S.K. and Chaudhuri, K. (2013) 'An optimal replenishment policy for EOQ models with time-varying demand and shortages', *International Journal of Services and Operations Management*, Vol. 16, No. 4, pp.443–459, DOI: 10.1504/IJSOM.2013.057508.

Sanni, S.S. and Chukwu, W.I.E. (2013) 'An Economic order quantity model for Items with three-parameter Weibull distribution deterioration, ramp-type demand and shortages', *Applied Mathematical Modelling*, Vol. 37, No. 23, pp.9698–9706, DOI: 10.1016/j.apm.2013.05.017.

Sicilia, J., González-De-la-Rosa, M., Febles-Acosta, J. and Alcaide-López-de-Pablo, D. (2014) 'Optimal policy for an inventory system with power demand, backlogged shortages and production rate proportional to demand rate', *International Journal of Production Economics*, Vol. 155, No. 1, pp.163–171, doi: 10.1016/j.ijpe.2013.11.020.

Skouri, K. and Konstantaras, I. (2009) 'Order level inventory models for deteriorating seasonable/fashionable products with time dependent demand and shortages', *Mathematical Problems in Engineering*, No. 1, pp.1–24, Article ID 679736, doi: 10.1155/2009/679736.

Taleizadeh, A.A. and Nematollahi, M. (2014) 'An inventory control problem for deteriorating items with back-ordering and financial considerations', *Applied Mathematical Modelling*, Vol. 38, No. 1, pp.93–109, DOI: 10.1016/j.apm.2013.05.065.

Taleizadeh, A.A., Niaki, S.T.A., Shafii, N., Meibodi, R.G. and Jabbarzadeh, A. (2010) 'A particle swarm optimization approach for constraint joint single buyer-single vendor inventory problem with changeable lead time and (r, Q) policy in supply chain', *The International Journal of Advanced Manufacturing Technology*, Vol. 51, Nos. 9–12, pp.1209–1223, DOI: 10.1007/s00170-010-2689-0.

Tyagi, A.P., Pandey, R.K. and Singh, S. (2014) 'An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and variable holding cost', *International Journal of Operational Research*, Vol. 21, No. 4, pp.466–488, DOI: 10.1504/IJOR.2014.065614.

Wang, S.P. (2002) 'On inventory replenishment with non-linear increasing demand', *Computers & Operations Research*, Vol. 29, No. 13, pp.1819–1825, DOI: 10.1016/S0305-0548(01)00060-0.

Wee, H.M. (1995) 'A deterministic lot-size inventory model for deteriorating items with shortages and a declining market', *Computers & Operations Research*, Vol. 22, No. 3, pp.345–356, DOI: 10.1016/0305-0548(94)E0005-R.

Yang, J., Zhao, G.Q. and Rand, G.K. (2004) 'An eclectic approach for replenishment with non-linear decreasing demand', *International Journal of Production Economics*, Vol. 92, No. 2, pp.125–131, DOI: 10.1016/j.ijpe.2003.09.017.

Yang, P.C., Wee, H.M. and Hsu, P.H. (2008) 'Collaborative vendor-buyer inventory system with declining market', *Computers & Industrial Engineering*, Vol. 54, No. 1, pp.128–139, DOI: 10.1016/j.cie.2007.06.041.

Yang, P.C., Wee, H.M., Chung, S.L. and Huang, Y.Y. (2013) 'Pricing and replenishment strategy for a multi-market deteriorating product with time-varying and price-sensitive demand', *Journal of Industrial and Management Optimization*, Vol. 9, No. 4, pp.769–787, DOI: 10.3934/jimo.2013.9.769.

Zhao, G.Q., Yang, J. and Rand, G.K. (2001) 'Heuristics for replenishment with linear decreasing demand', *International Journal of Production Economics*, Vol. 69, No. 3, pp.339–345, DOI: 10.1016/S0925-5273(00)00078-5.

Zhou, Y.W., Lau, H.S. and Yang, S.L. (2004) 'A finite horizon lot-sizing problem with time-varying deterministic demand and waiting-time-dependent partial backlogging', *International Journal of Production Economics*, Vol. 91, No. 2, pp.109–119, DOI: 10.1016/j.ijpe.2003.07.004.