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SESSION D2S2R6

A Heuristic Based on the Reduction Cost Concept for SFI Policy with Nonlinear Increasing Demand Pattern

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Abstract. *Traditional inventory policy EOQ has been developed in the past for constant demand pattern under infinite planning horizon. However, at the growth stage of product life cycle, demand rate for any particular products are not stable anymore but have increasing form. In that situation, thus, the use of EOQ might provide inappropriate result. In this paper, a heuristic technique based on the reduction cost concept for SFI (Shortage Followed by Inventory) policy with nonlinear increasing demand pattern under finite planning horizon is developed. In the proposed heuristic technique, first, Wang's consecutive method (Computer and Operations Research, 29:1819-1825, 2002) is used to determine shortage starting points. Then, the concept of reduction cost, which is defined as the difference between reduction of holding cost when shortage is allowed and the shortage cost that is incurred, is introduced and is applied to find replenishment times in each cycle. It is noted that the proposed heuristic technique is developed in such a way that demand during pre-defined planning horizon can always be fulfilled. Numerical examples is conducted to illustrate the applicability of the proposed technique and to compare the results from the proposed technique with those from other techniques developed in the past.*

Keywords: *Inventory policy, heuristic, nonlinear increasing demand, reduction cost concept, shortage*

1. INTRODUCTION

The stable demand pattern usually typifies for any particular products that have reached the mature stage of their life cycle. The popular EOQ policy can then be applied appropriately in this situation, as it was developed under the assumption of stable demand pattern. However, especially new product usually passes through a growth stage before reaching its mature stage. The growth stage itself is characterized by increasing demand pattern, thus, EOQ policy might not appropriate to be applied in this situation. A proper inventory policy for increasing demand pattern is then needed.

Initial works on inventory policy for linear increasing demand pattern were done by Resh et al.(1976) and Donaldson (1977) who developed the exact solution procedure, followed by other researchers such as Ritchie (1984), Hariga (1993), and Lo et al. (2002).

Due to the complexity of some exact solution procedures, many researchers have also developed the heuristic technique to solve inventory replenishment problem for linear increasing demand such as: Silver (1979); Phelps (19 Mitra et al., (1984); Teng (1994). The performance of th heuristics techniques above is comparable as they provide total inventory cost that is just slightly higher t that of the exact solution procedure of Donaldson (1977).

As in some products, the nonlinear form is more suita to represent the demand pattern; other researchers h developed also the inventory policy for this case. Unlike the case of linear increasing demand, there are only researchers have been found such as Yeng et al. (1999) Wang (2002). It is noted that Wang (2002) has introduced concept of reduction cost and has developed the consecut method that outperforms the best heuristic method provi by Yang et al. (1999).

All the researches that have been explained abo consider non-shortage case, which means that there always goods available to meet all the demand. However, reality, it happens a situation in which when custome demand arrives there are no stock of goods available, or called shortage. If all customers during this situation intending to wait until next replenishment arrived, then i

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† : Corresponding Author

called completely backlogged.

According to Zhou et al. (2004), the inventory policy for shortage case can be divided in two: 1) Inventory Followed by Shortage (IFS) and 2) Shortage Followed by Inventory (SFI). The research on IFS policy for nonlinear increasing demand and completely backlogged has been done by Hariga (1994) who developed exact solution procedure, followed by Yang (2006) who developed the exact solution procedure based on backward recursive algorithm. It is noted that Yang (2006) also provided the exact solution procedure not only [illegible] comprehensive mathematical effort is needed and the method can be applied only for a very specific demand pattern.

Beside exact solution, the heuristic method has been developed by Yang et al. (2002) who introduced the forward recursive algorithm for SFI policy with completely backlogged. It is noted that even though shortages are assumed to be completely backlogged there still exist shortages at the end of planning horizon, which means that demand over pre-defined planning horizon are not fulfilled. However, both of Yang et al. (2002) and Yang (2006) have not mentioned yet how to cope with this situation.

Another heuristic technique is recently done by Astanti & Luong (2008) who developed the heuristic technique for IFS policy with completely backlogged, in such a way that demand during pre-defined planning horizon can always be met. In the work of Astanti & Luong, they extended the work of Wang (2002) who was originally developed for non-shortage case and then compared the results with those from Nelder-Mead technique provided by Chen et al. (2007). It is noted that, Chen et al. (2007) proposed a direct search method based on the Nelder-Mead algorithm for IFS policy by considering whole stages of product life cycle, in which demand is assumed to follow a revised Beta function.

The focus of this research is to develop an algorithm for SFI policy with nonlinear increasing demand and completely backlogged in such a way that demands during pre-defined planning horizon can be met. To illustrate the applicability and the performance of the proposed technique, the results from it are then compared with those from Yang et al. (2002) and Chen et al. (2007). As originally Chen et al. (2007)'s method has been developed for IFS policy, then in this paper it is slightly modified to deal with SFI policy.

There are two stages used in the proposed technique. The first stage is purposed to find shortage point $\{s_i\}$ in a pre-defined planning horizon by applying the method of Wang's that originally was developed to find replenishment time $\{t_i\}$. Then, the technique based on the reduction cost concept is introduced and is applied to find replenishment time $\{t_i\}$ in each cycle. It is noted that by applying the proposed heuristic technique, demand over pre-defined planning horizon can always be met.

The remaining parts of this paper are organized as follows. Mathematical development, in which the expression of total inventory cost is developed, is presented in Section 2. In Section 3, the consecutive method of Wang (2002) to find shortage point $\{s_i\}$ and the proposed heuristic technique to find replenishment time $\{t_i\}$ is discussed. Some numerical experiments to illustrate the applicability of the proposed technique are conducted in Section 4, followed by some concluding remarks in Section 5.

2. MATHEMATICAL MODEL

The following notations are used throughout the paper:

H	:	length of planning horizon under consideration
$f(t)$	:	demand rate at time t, which is assumed to be a nonlinear increasing
c_1	:	ordering cost per order
c_2	:	holding cost per unit per unit time
c_3	:	shortage cost per unit per unit time
n	:	number of replenishment cycles in the planning horizon
t_i	:	the i^{th} replenishment time $(i = 1,2,...,n)$
s_i	:	the i^{th} shortage starting point $(i = 1,2,...,n,n+1)$, which is also the starting point of the i^{th} cycle $[s_i, s_{i+1}]$, except that $s_{n+1} = H$
$I(t)$	:	inventory level at time t, that is evaluated after replenishment arrives at time $t = t_i$ in the i^{th} cycle $[s_i, s_{i+1}]$

The behavior of the inventory level function under SFI policy is illustrated in Figure 1. For mathematical model development, the following assumptions are used:

i) Replenishments are made only at times t_i's ($i = 1,2,...,n$).
ii) Lead time is negligible, i.e., replenishment is instantaneous.
iii) The quantity replenishment that is received at time t is used to meet the accumulated shortage from time s_i to t_i $(s_i < t_i)$; $(i = 1,...,n)$
iv) Shortages are permitted at the beginning of each cycle but no shortages are permitted at the end of planning horizon (i.e., $s_{n+1} = H$).

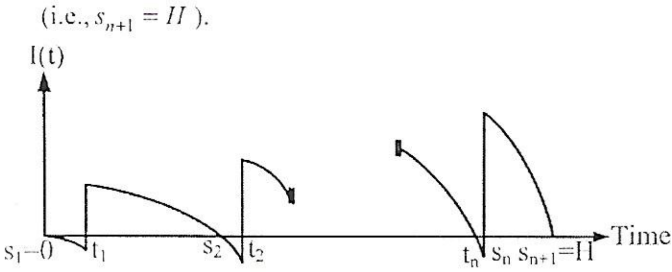

Figure 1: Inventory level under SFI policy

By using the assumption above, the expression for the total inventory cost; which comprises of ordering cost, holding cost and shortage cost; of SFI system during planning horizon H when n orders are placed is expressed below:

$$C(n,\{s_i\},\{t_i\}) = nc_1 + c_2\sum_{i=1}^{n} I_i + c_3\sum_{i=1}^{n} S_i \quad (1)$$

in which

I_i cumulative holding inventory during cycle i

S_i cumulative shortage during cycle i

The expressions of cumulative holding inventory I_i and cumulative shortage S_i for each cycle i from s_i to s_{i+1} will be derived in section 2.1 and 2.2 below.

2.1 Cumulative Holding Inventory I_i

Cumulative holding I_i in cycle i $(i = 1,2,...,n)$ can be determined as:

$$I_i = \int_{t_i}^{s_{i+1}}\int_{t}^{s_{i+1}} f(\tau)\,d\tau dt$$

$$= (s_{i+1} - t_i)F(s_{i+1}) - \int_{t_i}^{s_{i+1}} F(t)\,dt \quad (2)$$

in which

$F(t) = \int_0^t f(t)\,dt$, denote the cumulative demand from time 0 to time t.

2.2 Cumulative Shortage S_i

Cumulative shortage S_i in cycle i $(i = 1,2,...,n)$ can be determined as:

$$S_i = \int_{s_i}^{t_i}\int_{s_i}^{t} f(\tau)\,d\tau dt$$

$$= (s_i - t_i)F(s_i) + \int_{s_i}^{t_i} F(t)dt \quad (3)$$

From (2) and (3) the expression of the total inventory cost can be defined as:

$$C(n,\{s_i\},\{t_i\}) = nc_1 + c_2\sum_{i=1}^{n}\left[(s_{i+1} - t_i)F(s_{i+1}) - \int_{t_i}^{s_{i+1}} F(t)dt\right]$$
$$+ c_3\sum_{i=1}^{n}\left[(s_i - t_i)F(s_i) + \int_{s_i}^{t_i} F(t)dt\right] \quad (4)$$

3. HEURISTIC TECHNIQUE BASED ON THE CONCEPT OF REDUCTION COST

From (4), it can be seen that the total inventory cost function can be determined if we know the value of replenishment time t_i's and shortage points s_i's. In this paper, the value of shortage point s_i's is determined by using consecutive method proposed by Wang (2002), who originally developed it to find replenishment time t_i's for non-shortage case. Then, the proposed heuristic technique is applied to determine replenishment time t_i's. It is noted that the main idea of both Wang's consecutive method and the proposed heuristic technique is to check if there are any possibilities to reduce the total cost. Detail procedures to find shortage point s_i's and replenishment time t_i's are explained in Section 3.1 and 3.2 below.

3.1 Procedure to find s_i

As it was mentioned before, Wang's consecutive method was originally developed to find replenishment time t_i's for inventory policy with nonlinear increasing demand without shortage. However, in this paper, Wang's consecutive method is applied to help determine the shortage points s_i's.

In Wang's method, the approach begins with the use of one replenishment to meet demands over planning horizon H. Order size Q, therefore, will be cumulative demand for period $[0,H]$. The inventory level in this situation is presented in Figure 2.

To examine if there are any possibilities to reduce total inventory cost, another replenishment is placed at time t_2. Therefore, the order size of two replenishments, i.e., Q_1 and Q_2 are determined as the cumulative demand from period $[0,t_2]$ and $[t_2,H]$ respectively. It is noted that an additional replenishment can help to reduce inventory holding cost, but with the additional ordering cost will be added (see Figure 3). Hence, an additional replenishment t_2 is preferred if the reduction in holding cost is greater than an additional ordering cost. The optimal value of t_2 itself, if exist, can be found by maximizing the difference between reduction in holding cost and ordering cost. The same step can be then applied in each cycle until no reduction in total inventory cost has been found.

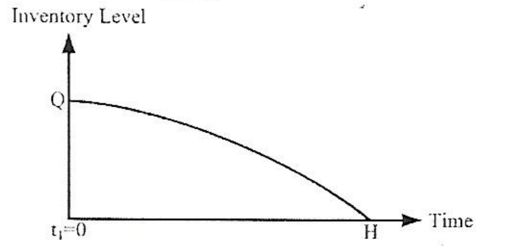

Figure 2: Inventory level with one replenishment

As the values of s_i and s_{i+1} are known before, then the value of $F(t_i)$ can be determined by using any search algorithm i.e. bisection algorithm, then the unique solution of t_i^* can be determined.
As the demand rate $f(t)$ is assumed to be increasing and $c_3 > c_2$, the second-order derivative is :

$$\left.\frac{d^2RC}{dt_i^2}\right|_{t_i=t_i^*} = -(c_2+c_3)f(t_i^*) < 0,$$

which implies that we can reach the maximum point at $t_i = t_i^*$.

4. Numerical Examples

In this section, numerical experiments are conducted to illustrate the applicability of the proposed technique. Two examples are considered here.

Example 1 (Yang et al.,2002): Consider demand function of the form: $f(t) = (a+bt)^u$ with $u = 2, a = 10, b = 30$. Other parameters are set as follows: $H = 1, c_1 = 4.5, c_2 = 1, c_3 = 3.5$.

Step-by-step procedure to determine s_i's $(i = 1,2,...,n)$ and t_i's $(i = 1,2,...,n)$ and is determined through a two-stage procedure as presented below.

Stage 1. Apply Wang's procedure as discussed in Section 3.1 to find shortage starting points. The details are explained as follows:

- At first, only one replenishment at time $t_1 = 0$ with order size Q_1^0 is considered to meet the demand over planning horizon $[0, H]$. The result is $Q_1^0 = 700$.
- In the first iteration, an additional replenishment at time $t_2 \in (0, H)$ is considered. The optimal value of t_2 can be found by maximizing the difference between the holding cost reduction and the ordering cost. The result is $t_2^* = 0.5988$ and the corresponding reduction cost is 280.3220, which is greater than the ordering cost. Therefore, this value of t_2^* is acceptable. The existence of t_2^* will then form two replenishment cycles in the planning horizon, i.e., $\left[0, t_2^*\right]$ and $\left[t_2^*, H\right]$.
- Next, we continue to examine in each of the above two replenishment cycles to determine if additional replenishment times $t_3 \in \left(0, t_2^*\right)$ and $t_4 \in \left(t_2^*, H\right)$ will help to reduce total inventory cost further. The same procedure is employed to find the optimal values of t_3, t_4 and the results are $t_3^* = 0.3501$ and $t_4^* = 0.8151$. The corresponding reduction costs are 51.5247 and 55.5379 respectively, which are greater than the cost of an additional order. Therefore, these two additional replenishments are also accepted.
- The next iteration with the same procedure is performed until no further improvement in total inventory cost is found. The complete result from Wang's procedure is presented in Figure 6a, in which the length of cycle i is from $\left[t_i, t_{i+1}\right]$. But, as in this paper we are dealing with the SFI policy in which the length of cycle i is from $\left[s_i, s_{i+1}\right]$, then the result from Wang's procedure is adjusted as is presented in Figure 6b. By this adjustment, then, the shortage starting point can be determined. The next stage is performed to determine replenishment time in each cycle i $\left[s_i, s_{i+1}\right]$.

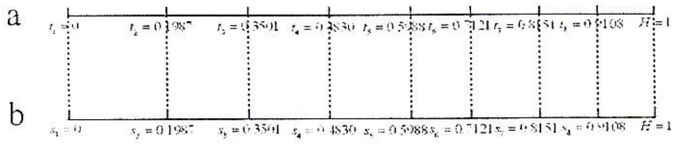

Figure 6: a Length of cycle i of Wang's.
b Length of cycle i of SFI policy.

Stage 2. In this stage, the proposed procedure that has been explained before in Section 3.2 is performed to determine optimal replenishment time in each cycle i $\left[s_i, s_{i+1}\right]$. It is noted that the optimal replenishment time t_i's can be found by solving maximization problem with the objective function is to maximize the reduction cost (the difference between the holding cost when shortages are allowed and the shortage cost that is incurred). Complete solutions for Example 1 is presented in Table 1 with the corresponding total inventory cost is $C = 67.46$.

Table 1: Optimal replenishment schedule – Example 1

Cycle i	s_i	t_i	Order Size
1	0.0000	0.0630	34.0559
2	0.1987	0.2396	50.5983
3	0.3501	0.3840	67.4477
4	0.4830	0.5115	79.7241
5	0.5988	0.6263	99.8823
6	0.7121	0.7368	111.5424
7	0.8151	0.8377	123.3745
8	0.9108	0.9317	133.3748
Total			700.000
Ending time	1.0000		

It is noted that the total inventory cost reported by Yang et al. (2002) for this example is 66.13 with $n = 7$. However, in the replenishment schedule reported by Yang et al. (2002),

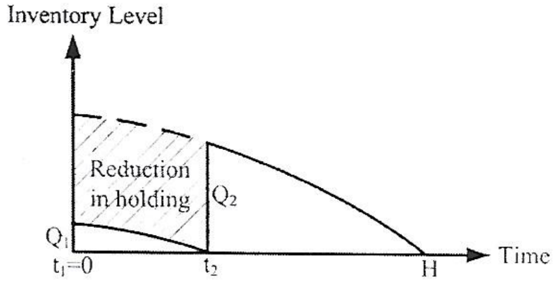

Figure 3: Inventory level with two replenishments

The optimal timing of replenishment resulted from Wang's is depicted in Figure 4a, with the length of cycle i is from $[t_i, t_{i+1}]$. As for SFI policy that is considered in this paper the length of cycle i goes from $[s_i, s_{i+1}]$, therefore, Figure 4a is adjusted to Figure 4b. Then, it can be clearly seen in Figure 4b now that the length of cycle i is from$[s_i, s_{i+1}]$. Thus, the shortage point that is defined as the point where the inventory drops to zero can easily be known. The next step is to determine the timing of replenishment t_i in each cycle $i\,[s_i, s_{i+1}]$.

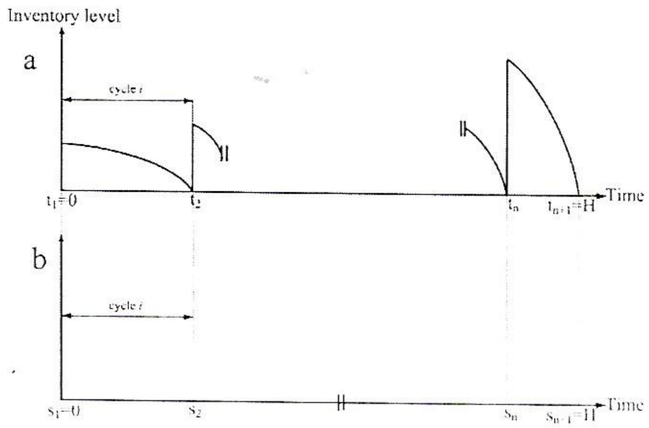

Figure 4: a Length of cycle i of Wang's.
b Length of cycle i of SFI policy.

3.2 Procedure to find t_i

Replenishment times t_i 's in each cycle i can be determined by solving a maximization problem, with the objective function is to maximize the reduction cost. Reduction cost here is defined as the difference between reduction in holding cost when shortage is allowed and the shortage cost that is incurred. The maximization problem is formulated as follows:

$$\text{Maximize } RC^{(i)} = c_2\left(A_1^{(i)}\right) - c_3\left(A_2^{(i)}\right) \qquad (5)$$

Subject to

$$s_i \le t_i \le s_{i+1}$$

in which:

$RC^{(i)}$ Reduction cost in cycle i

$A_1^{(i)}, A_2^{(i)}$ The areas showed in Figure 5, which represent fc the reduction of cumulative holding inventor and cumulative shortage in cycle i

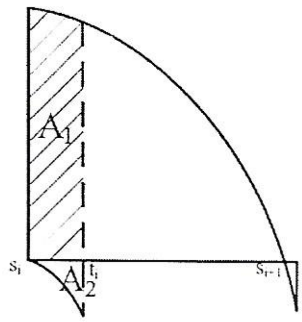

Figure 5: Reduction cost

Positive value of reduction cost means that shortage i allowed to be occurred in cycle i. Otherwise, shortage is nc allowed to be occurred, as the negative value of reductio cost $RC^{(i)}$ means that the appear of shortage will increas the total inventory cost. The expression of $RC^{(i)}$ i presented as follows.

$$RC^{(i)} = c_2 A_1^{(i)} - c_3 A_2^{(i)}$$

$$= c_2 \int_{s_i}^{t_i} \int_{t}^{s_{i+1}} f(\tau)d\tau dt - c_3 \int_{s_i}^{t_i} \int_{s_i}^{t} f(\tau)d\tau dt =$$

$$= c_2 \left\{ (t_i - s_i)F(s_{i+1}) - \int_{s_i}^{t_i} F(t)dt \right\}$$

$$+ c_3 \left\{ (t_i - s_i)F(s_i) - \int_{s_i}^{t_i} F(t)dt \right\}$$

$$= c_2(t_i - s_i)F(s_{i+1}) + c_3(t_i - s_i)F(s_i)$$

$$-(c_2 + c_3)\int_{s_i}^{t_i} F(t)dt \qquad (6)$$

If t_i^* is the optimal time for replenishment time durin interval $[s_i, s_{i+1}]$, then t_i^* satisfies the condition:

$$\frac{dRC}{dt_i} = 0$$

$$c_2 F(s_{i+1}) + c_3 F(s_i) - (c_2 + c_3)F(t_i) = 0 \qquad (7)$$

although shortages were assumed to be completely backlogged, there still existed a shortage period at the end of the planning horizon, and the author did not mention how to deal with this shortage (see Figure 7 for illustration).

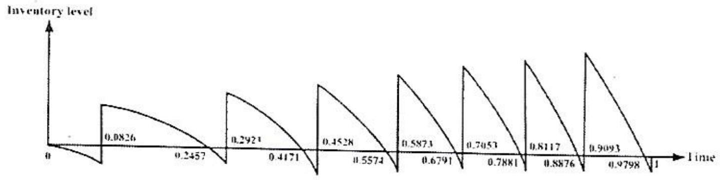

Figure 7: Optimal replenishment schedule of Yang et al. (2002)

Practically, there are two ways to fulfill the demand at the end of the planning horizon in this case either by (1) adding one replenishment at the end of the planning horizon with the replenishment quantity exactly equals to the shortage amount, or (2) increasing the time coverage of the last replenishment cycle. The associated costs of these two adjustments are shown also in Table 2 for comparison purpose.

Table 2: Comparison of the results between the proposed technique and adjusted Yang's et al. (2002)

Cycle i	Proposed technique		Adjusted Yang (1)		Adjusted Yang (2)	
	s_i	t_i	s_i	t_i	s_i	t_i
1	0.0000	0.0630	0.0000	0.0826	0.0000	0.0826
2	0.1987	0.2396	0.2457	0.2923	0.2457	0.2923
3	0.3501	0.3840	0.4171	0.4528	0.4171	0.4528
4	0.4830	0.5115	0.5574	0.5873	0.5574	0.5873
5	0.5988	0.6263	0.6791	0.7053	0.6791	0.7053
6	0.7121	0.7368	0.7881	0.8117	0.7881	0.8117
7	0.8151	0.8377	0.8876	0.9093	0.8876	0.9093
8	0.9108	0.9317	0.9798			
End time	1.0000		1.0000		1.0000	
C	67.46		70.63		67.58	

From Table 2, it can be seen that, if the total demand in the planning horizon must be strictly fulfilled, the total inventory cost resulted from our proposed technique is smaller as compared with the ones received from adjusted Yang's models.

Example 2 (Yang et al., 1999): In this example, the second example from 12 Sample Problems of Yang et al.(1999) is considered. Demand rate has a form $f(t)=a+bt+ct^2$, $(a,b,c\geq 0)$ with the detail parameters as follows: $a=0$, $b=900$, $c=100$, $H=2$, $c_1=9$, $c_2=2$.

The second sample problem taken from Yang, et al. (1999) is then solved by using our proposed technique. For comparison purpose, the second sample problem taken from Yang et al. (1999) is also solved by using Nelder-Mead technique provided by Chen et al. (2007), and the results are presented below in Table 3.

It is noted that the Nelder-Mead technique that has been proposed by Chen et al. (2007) originally was used to solve inventory replenishment problem for IFS policy by considering a whole stage of product life cycle. As the Nelder-Mead algorithm is a general search technique, the proposed technique of Chen et al. (2007) can be applied also for the case of increasing demand pattern. Therefore, in this paper, the Nelder-Mead technique provided by Chen et al. (2007) has been slightly modified to incorporate increasing demand pattern and SFI policy as well.

Table 3: Comparison of the results from proposed technique with those from Nelder-Mead technique – Example 2

Case	Proposed technique			Nelder-Mead technique		
	n	Total cost	Comp. Time* (second)	n	Total Cost	Comp. Time* (second)
$c_3=2.5c_2$	22	326.07	0.30	17	320.59	2539.70
$c_3=5c_2$	22	348.63	0.23	17	346.85	4348.90
$c_3=7.5c_2$	22	358.11	0.22	17	358.53	5842.09
$c_3=75c_2$	22	379.16	0.20	20	379.71	3319.80

(*: recorded on a PC with Intel P4 3.4GHz & 1 GB RAM)

From the results that are presented in Table 3, it can be seen that our proposed technique is comparable with the proposed technique of Chen et al. (2007) in term of solution quality. Moreover, the computational times required in our technique are much smaller.

5. CONCLUSION

In this proposed heuristic technique, the consecutive method of Wang (2002), who originally is developed to determine replenishment time for non shortage case, is used to determine the length of cycle $i\,[s_i, s_{i+1}]$.

The technique developed in this paper gives a new way to solve the inventory problem for nonlinear increasing demand pattern with SFI policy, in which shortage is allowed to occur in the beginning of cycle. From the numerical experiments results conducted in this paper, it can be seen that the total inventory cost resulted from the proposed technique is comparable with the result from adjusted Yang's et al. (2002). In addition, the proposed heuristic technique requires less computational time as compared with the method based on

Nelder-Mead of Chen et al. (2007).

The main contribution of this research is the new concept of reduction cost used in the proposed technique, which is easy for practitioners to understand and employ. Moreover, the proposed technique can always guarantee that the total length of replenishment intervals always match exactly with the planning horizon which means that demand over pre-defined planning horizon can always be met. However, it should be noted that the proposed technique here can only help to deal with the cases of increasing demand and completely backlog.

Further development of this technique is possible to be done i.e., by considering partial backlog case.

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