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Heuristic Algorithm Based on Consecutive Method for Inventory Policy with Partial Backlog Case and Deterministic Demand

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ABSTRACT

Originally, the consecutive method was proposed by Wang [1] to determine inventory policy for the non shortage case. Later on Astanti and Luong [2] extended the work of Wang by developing new algorithm based on consecutive method for shortage case but limited on completely backlog situation. It is noted that the shortage case is the situation when customer arrive but the goods are not there. When all customers are willing to wait until the next replenishment, it is called completely backlog situation. However, in the real situation it often happens that some customers are willing to wait, but the other prefers to find another supply sources, or it is called partial backlog case. The work of this research, therefore, is going to develop the algorithm for determining inventory policy with partial backlog case.

Keywords: *Inventory policy, Heuristic, Consecutive method, Partial backlog*

1. Introduction

Inventory policy deals with "*management of stock level of goods*" [3] and comprises of decisions about when and how much the order/production should be placed. Those decisions are crucial, since a company have to manage its inventory properly in order to meet customer's demands with minimum cost.

Before developing an inventory policy, demand should firstly be estimated. Demand of a particular products itself has either probabilistic or deterministic characteristic. Many researches on inventory policy have been done dealing with probabilistic demand such as [4-12], among others. However, the focus of this research is on deterministic demand. It is noted that if the result of the developed algorithm is promising for the case of deterministic demand, then the developed algorithm will be extended to the case of probabilistic demand.

In reality, the shortage period might occur in each replenishment cycle, e.g., when customer arrives, no stocks of goods are available. From the standpoint of supplier, shortage sometimes is economically preferable, e.g., when holding cost is significant high as compared with shortage cost. The shortage case can be divided into two which are completely backlog and partial backlog cases. The completely backlog case is the situation when customer arrive but the goods are not there, but all customers are willing to wait until the next replenishment. In the partial backlog case, however, some customers are willing to wait, but the other prefers to find another supply sources.

Inventory models with infinite planning horizon of partial backlog case with fixed fraction of backlog had been proposed in [13]. They developed models and solution technique for not only constant demand rate, but also stochastic demand rate, in which covers continuous and periodic review models. The work had been extended by considering customer impatience into the fraction of backlogged demand, in which the backlog fraction is modeled as linear function [14] and general non increasing function [15]. Instead of infinite planning horizon, a finite planning horizon model for this situation [16] developed the heuristic method for SFI policy, not only for linear increasing but also for nonlinear increasing demand.

Based on the research that was done by [2], it shows that the results of heuristic algorithm based consecutive method are comparable to the results of other methods, especially for the case of deterministic demand and completely backlog case. More over that method is easy in concept and computationally simple. The work of this research, therefore are going to develop the algorithm for determining inventory policy with partial backlog case and finite planning horizon. Section 2 of this paper is containing the mathematical model of the inventory model with finite planning horizon of partial backlog case with fixed fraction backlog. Then, the heuristic algorithm based on consecutive method for solving the problem is presented in Section 3. Finally, an example of the application of the heuristic is presented in Section 4.

2. Mathematical Model

The following assumptions are used throughout the paper:

H length of the planning horizon under consideration
$f(t)$ instantaneous demand rate at time t
c_{1f} fixed ordering cost per order
c_{1v} variable purchasing cost per order
c_2 holding cost per unit per unit time
c_{3s} the backlogging cost per unit per unit time due to shortages
c_{3l} the unit cost of lost sales
n number of replenishment over $[0,H]$
t_i the ith replenishment time $(i=1,2,...,n)$
s_i the shortage point at cycle i, which is the time at which the inventory level reaches zero in the ith cycle $[s_i,s_{i+1}]$; except that $(s_{n+1}=H)$
$I(t)$ inventory level at time t, that is evaluated after replenishment arrives at time $t=t_i$ in the ith cycle $[s_i,s_{i+1}]$

The behavior of inventory level function of the inventory problem that is considered in this paper is presented in Figure 1.

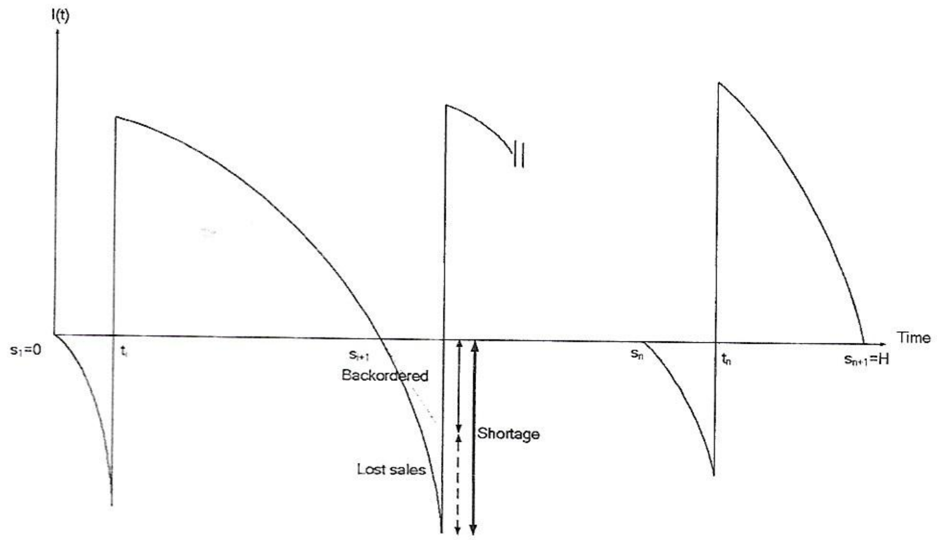

Figure 1. Inventory level over the whole planning horizon

For the development of mathematical model, the following assumptions are used:

a) The mixture of backorders and lost sales during the stock out period is known and constant. p is the probability that an order will be backlog when shortage occurs, therefore the fraction of lost sales is $(1-p)$.
b) The replenishment is made at time t_i $(i=1,2,...,n)$
c) The quantity received at t_i is used partly to meet accumulated backordered in the cycle from time s_i to t_i $(s_i<t_i)$
d) The inventory at t_i gradually reduces to zero at s_{i+1} $(s_{i+1}>t_i)$, in which a new cycle starts.
e) Shortages are permitted in the beginning of the period ($s_1=0$) but no shortages are permitted at the last cycle ($s_{n+1}=H$)

Using the assumptions above, the expression for the total cost which includes ordering cost, variable purchasing cost, holding cost, backlog cost, and cost of lost sales, total cost of the inventory system during the planning horizon H when n order are placed is as follows:

$$C\left(n,\{s_i\},\{t_i\}\right)=\sum_{i=1}^{n}P_i+c_2I_i+c_{3s}S_i+c_{3l}L_i \qquad (1)$$

in which

I_i is the inventory level during ith cycle

$$I_i=\int_{t_i}^{s_{i+1}}\int_{t}^{s_{i+1}}f(\tau)d\tau dt=\int_{t_i}^{s_{i+1}}(t-t_i)f(t)dt \qquad (2)$$

P_i is the purchase cost during the ith replenishment cycle, and as:

$$P_i=c_{1f}+c_{1v}\left[\int_{si}^{ti}pf(t)dt+\int_{t_i}^{s_{i+1}}f(t)dt\right] \qquad (3)$$

S_i is the amount of backordered during ith cycle, as:

$$S_i=\int_{s_i}^{t_i}S(t)dt \qquad (4)$$

$$S_i=\int_{s_i}^{t_i}\int_{s_i}^{t}pf(\tau)d\tau dt=p\int_{s_i}^{t_i}(t_i-t)f(t)dt$$

L_i is the number of lost sales during ith cycle, as:

$$L_i=\int_{s_i}^{t_i}(1-p)f(t)dt \qquad (5)$$

From (2), (3), (4), and (5) the expression of the total cost can be defined as:

$$C\left(n,\{s_i\},\{t_i\}\right)=\sum_{i=1}^{n}c_{1f}+c_{1v}\left[\int_{si}^{ti}pf(t)dt+\int_{t_i}^{s_{i+1}}f(t)dt\right]+c_2\int_{t_i}^{s_{i+1}}(t-t_i)f(t)dt$$
$$+c_{3l}\int_{s_i}^{t_i}(1-p)f(t)dt+c_{3s}p\int_{s_i}^{t_i}(t_i-t)f(t)dt \qquad (6)$$

3. Consecutive Improvement Method for Partial Backlogging Case

From (6), it can be seen that the total inventory cost function can be determined if we know the value of replenishment time t_i 's and shortage points s_i 's. In this paper, the value of shortage point s_i 's is determined by using consecutive method proposed by Wang [1], who originally developed it to find replenishment time t_i 's for non-shortage case. Then, the proposed heuristic technique is applied to determine replenishment time t_i 's. It is noted that the main idea of both Wang's consecutive method and the proposed heuristic technique is to check if there are any possibilities to reduce the total cost. Detail procedures to find replenishment time t_i 's are explained in Section 3.1.

3.1. Procedure to find $\{t_i\}$

Replenishment times t_i 's in each cycle i can be determined by solving a maximization problem, with the objective function is to maximize the reduction cost. Reduction cost here is defined as the difference between the cost of holding the inventory with cost of backorder and lost sales. The reduction cost is presented graphically in Figure 2 and can be written as follows:

$$\text{Maximize } RC^{(i)}=c_2\left(\int_{s_i}^{t_i}\int_{t}^{s_{i+1}}f(t)dtdt\right)+\left(c_{1v}-c_{3l}\right)\left(\int_{s_i}^{t_i}(1-p)f(t)dt\right)-c_{3s}\left(\int_{s_i}^{t_i}\int_{s_i}^{t}pf(t)dtdt\right) \qquad (6)$$

Subject to

$$s_i\le t_i\le s_{i+1}$$

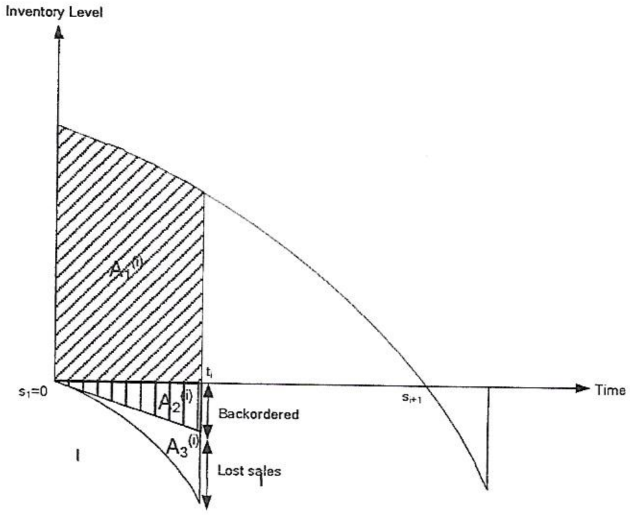

Figure 2. Reduction Cost

If the maximum reduction cost give the positive value it means that a replenishment is allowed to be occurred during the cycle i, otherwise a replenishment is not allowed to be occurred, since the negative value of reduction cost means that a replenishment will increase the total cost.

If t_i^* is the optimal time for replenishment time during the interval $[s_i, s_{i+1}]$, then t_i^* satisfies the Equation 7 :

$$\frac{dRC^{(i)}}{dt_i} = [c_2F(s_{i+1}) - c_2F(t_i) + c_{1v}(1-p)f(t_i)] - [c_{3l}(1-p)f(t_i) - c_{3s}pF(s_i) + c_{3s}pF(t_i)] = 0 \quad (7)$$

As the demand rate $f(t)$ is assumed to be increasing and $c_{3l} > c_{1v}$, the second-order derivative is :

$$\left.\frac{d^2RC^{(i)}}{dt_i^2}\right|_{t_i=t_i^*} = -c_2f(t_i) + (1-p)f'(t_i)(c_{1v} - c_{3l}) - c_{3s}pf(t_i) < 0,$$

which implies that we can reach the maximum point at $t_i = t_i^*$.

4. Numerical Examples

To illustrate the result, we apply the proposed methods to solve some numerical examples.

Example 1.

The example is similar with the example on [17], but in this paper no deterioration rate is applied, and the fraction of backorder is fixed ($p = 0.3$) and also in order to meet the assumption that $c_{3l} > c_{1v}$, therefore the value of c_{3l} and c_{1v} are changed.

Let $f(t) = 50 + 3t$, $H = 4$, $c_{1f} = 250$, $c_{1v} = 200$, $c_2 = 40$, $c_{3s} = 80$, $c_{3l} = 220$.

This example is then solved by the proposed method using Matlab 7.1, and the result is presented below in Table 1 with total cost of 48951.88

[12] Tarim, S.A., and Kingsman, B. G. (2006). Modelling and computing (R^n,S^n) policies for inventory systems with non-stationary stochastic demand. *European Journal of Operational Research,* 174(1), 581–599

[13] Pujawan, I.N., and Silver, E. A. (2008). Augmenting the lot sizing order quantity when demand is probabilistic. *European Journal of Operational Research,* 188(2), 705–722

[14] Montgomery, D. C., Bazaraa, M. S., & Keswani, A. K. (1973). Inventory model with a mixture of backorders and lost sales. *Naval Research Logistics,* 20(2), 255–263

[15] San Jose, L. A., Sicilia, J., and Garcia-Laguna, J. (2005). An inventory system with partial backlogging modeled according to a linear function. *Asia-Pacific Journal of Operational Research,* 22(2), 189–209

[16] San Jose, L. A., Sicilia, J., and Garcia-Laguna, J. (2005). The lot size-reorder inventory system with customers impatience functions. *Computers & Industrial Engineering,* 49, 349–362

[17] Zhou, Y. W., Lau, H. S., and Yang, S. L. (2004). A finite horizon lot-sizing problem with time-varying deterministic demand and waiting-time-dependent partial backlogging. *International Journal of Production Economics,* 91(2), 109–119

[18] Dye, C. Y., Chang, H. J., Teng, J. T. (2006) A deteriorating inventory model with time-varying demand and shortage-dependent partial backlogging. *European Journal of Operational Research,* 172 (2), 417–429.

[19] Chen, C.K., Hung, T.W., & Weng, T.C. (2007) Optimal replenishment policies with allowable shortages for a product life cycle. *Computers and Mathematics with Applications, 53,* 1582-1594.

Table 1. Replenishment Schedule for Example 1

cycle i	t_i	s_i
1	0.1578	0
2	0.6783	0.5238
3	1.1912	1.0398
4	1.6972	1.5487
5	2.1967	2.0507
6	2.6904	2.5471
7	3.1779	3.0371
8	3.6598	3.5214
9		4

Example 2.
The second example is applied for the case when the demand rate is constant.
In this case $f(t)=3t$ $H=4$, $c_{1f}=250$, $c_{1v}=200$, $c_2=40$, $c_{3s}=80$, $c_{3l}=220$.
This example is then solved by the proposed method using Matlab 7.1, and the result is presented below in Table 2 with total cost of **6126.14.**

Table 2. Replenishment Schedule for Example 2

cycle i	t_i	s_i
1	0.8004	0
2	1.7439	1.3333
3	2.6564	2.3094
4	3.4955	3.2041
5		4

5. Discussion and Further Work

Based on the numerical examples provided in the previous section, it can be seen that the proposed algorithm works for the case of linear increasing demand. Further work will be conducted to explore the application of the proposed method to more varying demand pattern. In addition, the proposed method will be compared to other heuristic method i.e. Nelder Mead technique proposed by Chen [18].

6. References

[1] Wang, S. P. (2002a). On inventory replenishment with non-linear increasing demand. *Computers and Operations Research*, 29(13), 1819–1825

[2] Astanti R. D., and Luong, H. T. (2009). A heuristic technique for inventory replenishment policy with increasing demand pattern and shortage allowance. *International Journal of Advanced Manufacturing Technology*, 41(11-12), 1199–1207

[3] Heyman, D. P., and Sobel, M. J. (1990). *Handbook in Operation Research and Management Science (Volume 2)*. Netherlands: Elsevier Science Publisher B.V.

[4] Silver, E. A. (1978). Inventory control under a probabilistic time-varying, deman

[5] d pattern. *AIIE Transaction,* 10(4), 371–379

[6] Askin, R. G. (1981). A procedure for production lot sizing with probabilistic dynamic demand. *AIIE Transaction,* 13(2), 132–136

[7] Wemmerlöv, U., and Whybark, D.C. (1984). Lot-sizing under uncertainty in a rolling schedule environment. *International Journal of Production Research,* 22(3), 467– 484

[8] Bookbinder, J. H., H'Ng, B-T. (1986). Rolling horizon production planning for probabilistic time-varying demands. *International Journal of Production Research,* 24(6), 1439–1458

[9] Vargas, V. A., and Metters, R. (1996). Adapting lot-sizing technique to stochastic demand through production scheduling policy. *IIE Transaction,* 28(2), 141–148

[10] Bollapragada, S., and Morton, T.E. (1999). A simple heuristic for computing non stationary (s,S) policies. *Operations Research,* 47(4), 576–584

[11] Tarim, S.A., and Kingsman, B. G. (2004). The stochastic dynamic production/inventory lot-sizing problem with service-level constraint. *International Journal of Production Economics* ,88(1), 105–119