An EPQ with Shortage Backorders Model on Imperfect Production System Subject to Two Key Production Systems

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Abstract This paper is an extension of the work of Lin and Gong (2011) on Economic Production Quantity (EPQ) model on an imperfect production system over infinite planning horizon, where the production system is dictated by two unreliable key production subsystems (KPS). While any shortage on the inventory of product was not allowed in the model of Lin and Gong (2011), planned shortage backorders is considered in the model proposed in this paper. The mathematical model is developed in order to determine production run time (τ) and production time when backorder is replenished (T_1) that minimizes the expected total cost per unit time including setup, inventory carrying, shortage, and defective costs. Approaches to solve the model are also being proposed in this paper, altogether with some numerical examples.

Keywords Economic production quantity model · Shortage backorders · Imperfect production system · Optimization technique · Approximation method

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1 Introduction

Productivity is generally defined as the ratio between output and input. The input can be man, material, machine, money, and method. In manufacturing industry, where the output is tangible product, the productivity can be measured by how many or how much the output resulted. Productivity might be affected by one of the input mentioned above, such as machine. Machine is one element of the production subsystem. The machine is reliable if it can perform as good as the standard. However, in reality there is a condition where, the machine does not perform well or it is called imperfect condition. This condition might happen due to, for example, machine breakdown. As illustration, boiler breakdown in a Crude Palm Oil (CPO) industry will increase the concentration of ALB in the oil so that it will decrease the quality of CPO (Sulistyo et al. 2012). Therefore, the output of the CPO is also decreased. In other word, the productivity of the industry is decreased.

The economic production quantity (EPQ) model developed by many researchers in the past, such as Silver et al. (1998) under the assumption the production subsystem is perfect (no breakdown). However, this ideal condition is rarely happened in the real situation. If this model is applied in the real situation where the production subsystem is imperfect, then, the target production is never be reached. The EPQ model considering imperfect production subsystem have been proposed by some researchers, such as Rosenblatt and Lee (1986). They assumed that in the production system there may exist an imperfect condition where the in-control state changed to out-control state where the random time to change is assumed following exponential distribution. As the result, the system might produce defective product. Following this work, some models dealt with various additional system setting had been proposed, such as (Lee and Rosenblatt 1987, 1988, 1989; Lee and Park 1991; Lin et al. 1991; Ben-Daya and Hariga 2000; Hsieh and Lee 2005). In those previous models, the imperfectness of production system is assumed to be dictated by single key production subsystem (KPS) only. Lin and Gong (2011) recently extended the study by proposing an EPQ model where the production system is imperfect and dictated by two imperfect KPS's over an infinite planning horizon. Ai et al. (2012) continued this work by considering finite planning horizon.

While those above mentioned researches discussed on EPQ model without shortage, the research conducted by Chung and Hou (2003) extended the work of Rosenblatt and Lee (1986) by incorporating the shortage into their model. Shortage itself can be defined as the situation when the customer order arrive but the finished good are not yet available. When all customer during the stockout period are willing to wait until the finished goods are replenished then it is called as completely backorder case. Shortage is common in practical situation, when the producer is realized that its customers loyal to their product.

This paper is extending the work of Chung and Hou (2003) and Lin and Gong (2011) by combining both works into a new EPQ model that consider 2 (two) imperfect KPS and shortage. The organization of this paper is as follow: Sect. 2

describes the mathematical model development, Sect. 3 discusses the solution methodology of the proposed method, Sect. 4 presents the numerical example, followed by some concluding remarks in Sect. 5.

2 Mathematical Model

This paper considers a production lot sizing problem where a product is to be manufactured in batches on an imperfect production system over an infinite planning horizon, in which shortage of product is allowed at the end of each production cycle and all shortage is backordered. The demand rate is d, and the production rate is p. As defined in Lin and Gong (2011) and Ai et al. (2012), the imperfectness of the system is shown on two imperfect key production subsystems (KPS) that may shift from an in-control to an out-of-control state due to three independent sources of shocks: source 1's shock causes first KPS to shift, source 2's shock causes second KPS to shift, and source 3's shock causes both KPS to shift. Each shocks occur at random time U_1, U_2, and U_{12} that follows exponential distribution with mean $1/\lambda_1$, $1/\lambda_2$, and $1/\lambda_{12}$, respectively. When at least one KPS on out-of-control state, consequently, the production system will produced some defective items with fixed but different rates: α percentage when first KPS out-of-control, β percentage when the second KPS out-of-control, and δ percentage when the both KPS out-of-control. The cost incurred by producing defective items when the first KPS is shifted, the second KPS is shifted, and both KPS are shifted are π_1, π_2, and π_{12}, respectively.

The production cycle of this situation can be described as Fig. 1, in which the inventory level is increased during the production uptime (τ) at rate ($p - d$) and decreased at rate $-d$ during the production downtime. It is shown in Fig. 1, the production cycle length T can be divided into four sections, each of them with length T_1, T_2, T_3, and T_4, respectively. The backorders are replenished in Sect. 1, in which the inventory level is increased from $-B_{max}$ to 0. The inventories are accumulated during Sect. 2, in which inventory level at the end of this section is $I_{\max}$. After that, the inventory level is decreased to 0 during Sect. 3 and the shortage happened in Sect. 4.

The optimization problem is to determining optimal production run time τ and production time when backorder is replenished T_1, that minimizes the expected total cost per unit time including setup, inventory carrying, shortage and defective costs.

In single production cycle, although shortages are exist, the number of product being produced ($p.\tau$) is equal to the demand of product ($d.T$) in that cycle. Therefore $T = p\tau/d$. If the setup cost is denoted as A, based on (1), the setup cost per unit time (C_1) can be defined as

$$C_1 = \frac{A}{T} = \frac{Ad}{p\tau} \tag{1}$$

The average inventory per production cycle as function of τ and T_1 can be expressed as

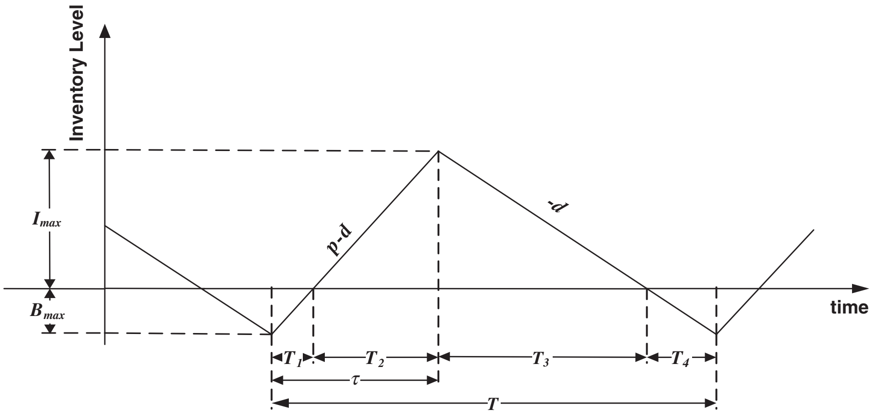

Fig. 1 Production cycle

$$\bar{I}(T_1, \tau) = \frac{(p-d)\tau}{2} - (p-d)T_1 + \frac{(p-d)T_1^2}{2\tau} \tag{2}$$

Therefore, if unit inventory holding cost per unit time is defined as h, the inventory carrying cost per unit time (C_2) can be defined as

$$C_2 = \frac{h(p-d)\tau}{2} - h(p-d)T_1 + \frac{h(p-d)T_1^2}{2\tau} \tag{3}$$

The average shortage per production cycle as function of τ and T_1 can be expressed as

$$\bar{B}(T_1, \tau) = \frac{(p-d)T_1^2}{2\tau} \tag{4}$$

Therefore, if unit shortage cost per unit time is defined as s, the shortage cost per unit time (C_3) can be defined as

$$C_3 = \frac{s(p-d)T_1^2}{2\tau} \tag{5}$$

The results from Lin and Gong (2011) are presented below to obtain the defective cost per unit time (C_4), in which the unit defective cost when the first KPS is shifted, the second KPS is shifted, and both KPS are shifted is defined as π_1, π_2, and π_{12}, respectively.

$$C_4 = \frac{d(\pi_1 E[N_1(\tau)] + \pi_2 E[N_2(\tau)] + \pi_{12} E[N_{12}(\tau)])}{p\tau} \tag{6}$$

$$E[N_1(\tau)] = p\alpha\left(\frac{1 - \exp[-(\lambda_2 + \lambda_{12})\tau]}{\lambda_2 + \lambda_{12}} - \frac{1 - \exp[-(\lambda_1 + \lambda_2 + \lambda_{12})\tau]}{\lambda_1 + \lambda_2 + \lambda_{12}}\right) \tag{7}$$

$$E[N_2(\tau)] = p\beta\left(\frac{1-\exp[-(\lambda_1+\lambda_{12})\tau]}{\lambda_1+\lambda_{12}} - \frac{1-\exp[-(\lambda_1+\lambda_2+\lambda_{12})\tau]}{\lambda_1+\lambda_2+\lambda_{12}}\right) \tag{8}$$

$$\begin{aligned} E[N_{12}(\tau)] = p\delta\Bigg(& \frac{\exp[-(\lambda_1+\lambda_{12})\tau]+(\lambda_1+\lambda_{12})\tau-1}{\lambda_1+\lambda_{12}} \\ & + \frac{\exp[-(\lambda_2+\lambda_{12})\tau]+(\lambda_2+\lambda_{12})\tau-1}{\lambda_2+\lambda_{12}} \\ & - \frac{\exp[-(\lambda_1+\lambda_2+\lambda_{12})\tau]+(\lambda_1+\lambda_2+\lambda_{12})\tau-1}{\lambda_1+\lambda_2+\lambda_{12}}\Bigg) \end{aligned} \tag{9}$$

Therefore, the expected total cost per unit time can be stated as

$$Z[\tau, T_1] = C_1 + C_2 + C_3 + C_4$$

$$\begin{aligned} Z[\tau, T_1] = \frac{Ad}{p\tau} & + \frac{h(p-d)\tau}{2} - h(p-d)T_1 + \frac{h(p-d)T_1^2}{2\tau} + \frac{s(p-d)T_1^2}{2\tau} \\ & + \frac{d(\pi_1 E[N_1(\tau)] + \pi_2 E[N_2(\tau)] + \pi_{12} E[N_{12}(\tau)])}{p\tau} \end{aligned} \tag{10}$$

3 Solution Methodology

Following Lin and Gong (2011), all exponential terms in the total cost expression can be approximate by MacLaurin series:

$$\exp(-\lambda\tau) \approx 1 - \lambda\tau + \frac{1}{2!}(\lambda\tau)^2 - \frac{1}{3!}(\lambda\tau)^3 \tag{11}$$

Therefore the total cost expression can be rewritten as

$$Z[\tau, T_1] \approx \frac{Ad}{p\tau} + h(p-d)\left[\frac{\tau}{2} - T_1\right] + \frac{(h+s)(p-d)T_1^2}{2\tau} + \frac{H\tau}{2} - \frac{K\tau^2}{6} \tag{12}$$

$$H = d(\pi_1\alpha\lambda_1 + \pi_2\beta\lambda_2 + \pi_{12}\delta\lambda_{12}) \tag{13}$$

$$K = d[\pi_1\alpha\lambda_1(\lambda_1 + 2\lambda_2 + 2\lambda_{12}) + \pi_2\beta\lambda_2(2\lambda_1 + \lambda_2 + 2\lambda_{12}) + \pi_{12}\delta(\lambda_{12}^2 - 2\lambda_1\lambda_2)] \tag{14}$$

It is well known from calculus optimization that the necessary condition for minimizing $Z[\tau, T_1]$ are the first partial derivatives equal to zero. Applying this condition for Eq. (12), it is found that

$$\frac{\partial}{\partial\tau} Z[\tau, T_1] = \frac{H}{2} + \frac{h(p-d)}{2} - \frac{K\tau}{3} - \frac{Ad}{p\tau^2} - \frac{(h+s)(p-d)T_1^2}{2\tau^2} = 0 \tag{15}$$

$$\frac{\partial}{\partial T_1} Z[\tau, T_1] = -h(p-d) + \frac{(h+s)(p-d)T_1}{\tau} = 0 \tag{16}$$

Solving Eq. (16) for T_1, it is found that

$$T_1^* = \frac{h}{(h+s)}\tau^* \tag{17}$$

Substituting Eq. (17) to Eq. (15), it is obtained that

$$\frac{H}{2} + \frac{hs(p-d)}{2(h+s)} - \frac{K\tau}{3} - \frac{Ad}{p\tau^2} = 0 \tag{18}$$

If the term $K\tau/3$ is neglected or approximated as zero, it found after some algebra that

$$\tau^* = \sqrt{\frac{2Ad}{p\left[H + hs\left(\frac{p-d}{h+s}\right)\right]}} = \sqrt{\frac{2Ad}{p\left[d(\pi_1\alpha\lambda_1 + \pi_2\beta\lambda_2 + \pi_{12}\delta\lambda_{12}) + hs\left(\frac{p-d}{h+s}\right)\right]}} \tag{19}$$

The sufficient condition of this result can be easily proven, since the Hessian matrix is positive definite.

4 Numerical Examples

To illustrate the proposed model and solution methodology, the numerical example is conducted on 8 (eight) sample problems as it is shown in Table 1.

Table 1 Parameters and solutions of sample problems

Parameters	Prob 1	Prob 2	Prob 3	Prob 4	Prob 5	Prob 6	Prob 7	Prob 8
d	200	200	200	200	200	200	200	200
p	300	300	300	300	300	300	300	300
α	0.1	0.1	0.3	0.3	0.1	0.1	0.3	0.3
β	0.1	0.1	0.3	0.3	0.1	0.1	0.3	0.3
δ	0.16	0.16	0.48	0.48	0.16	0.16	0.48	0.48
λ_1	0.05	0.05	0.05	0.05	0.15	0.15	0.15	0.15
λ_2	0.1	0.1	0.1	0.1	0.3	0.3	0.3	0.3
λ_{12}	0.02	0.02	0.02	0.02	0.06	0.06	0.06	0.06
π_1	10	10	10	10	10	10	10	10
π_2	10	10	10	10	10	10	10	10
π_{12}	12	12	12	12	12	12	12	12
A	100	100	100	100	100	100	100	100
h	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
s	0.16	0.24	0.16	0.24	0.16	0.24	0.16	0.24
Solutions								
τ^*	1.761	1.747	1.061	1.058	1.061	1.058	0.622	0.622
T_1^*	0.587	0.437	0.354	0.265	0.354	0.265	0.207	0.155
$Z[\tau^*, T_1^*]$	75.73	76.32	125.6	126	125.6	126	214.3	214.5

5 Concluding Remarks

This paper extend the work of Chung and Hou (2003) and Lin and Gong (2011) by incorporating 2(two) imperfect KPS considering shortage on EPQ model. Based on the numerical example it can be shown that the proposed model and its solution methodology works on 8 (eight) sample problems. Further work will be conducted to find another solution methodology approaches and to do the sensitivity analysis on the proposed model. In addition, formulating an EPQ model with 2(two) imperfect KPS considering shortage can be further investigated for finite planning horizon.

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