Paper 33

by The Jin Ai

A Joint Replenishment Inventory Model for Imperfect Quality Items with Shortages

The Jin Ai†, Ririn Diar Astanti
Department of Industrial Engineering
Universitas Atma Jaya Yogyakarta, Yogyakarta, Indonesia
Tel: (+62) 274-487711, Email: jinai@mail.uajy.ac.id, ririn@mail.uajy.ac.id

Dah-Chuan Gong
Department of Industrial and Business Management
Chang Gung University, Taoyuan, Taiwan
Email: gongdc@mail.cgu.edu.tw

Abstract. An n items joint replenishment inventory problem is considered here, where the demand of each items are constant and deterministic. A joint replenishment is conducted periodically every T time intervals. However, all items may not be included in each replenishment. Item i is only included every Z_iT time intervals. Replenishment of item are instantaneous and contain of imperfect quality items. After screening, items of poor quality are sorted, kept in stock and sold at a salvage value as a single batch within the replenishment cycle. Shortages are allowed and completely backordered with maximum backordering quantity B_i. Mathematical model is formulated in order to determining the basic time cycle T, replenishment multiplier Z_i, and backordering quantity B_i in order to maximize the expected total profit per unit time. A solution methodology is proposed for solve the model and a numerical example is provided for demonstrating the effectiveness of the proposed methodology.

Keywords: inventory model, joint replenishment, imperfect quality items, shortages.

1. INTRODUCTION

In order to obtain cost efficiency in the multi item inventory system, joint replenishment of items is one possible way to be conducted. In the inventory research literature, a problem so called the joint replenishment problem (JRP) has been investigated in order to address the situation in multi item inventory system where a group of items may be jointly ordered from a single supplier (Khouja and Goyal, 2008; Moon et al., 2008; Tsai et al., 2009; Wang et al., 2012)

Since assumption of perfect quality is unrealistic in the industrial application, nowadays there is a trend for relaxing the assumption of perfect quality in the inventory modeling research area. Salameh and Jaber (2000) proposed an economic production quantity model for a buyer who receives imperfect item from its supplier. After that, some other researchers have been extending their work by considering various operational setting in the inventory modeling (Konstantaras et al., 2007; Yoo et al., 2009; Chang and Ho, 2010). Shortage backorder also had been considered in some extension of this work, i.e. Rezaei (2005) and Wee et al. (2007).

In the JRP research area, only Paul et al. (2014) that has incorporated the relaxation of perfect quality items in the JRP model by considering the cases of with and without price discount. This paper is considering the case of shortage in the JRP model for imperfect quality items, that to the best of authors knowledge never been proposed before. A mathematical model is formulated for this situation and a solution methodology is proposed to solve the mathematical model.

The remainder of this paper is organized as follows: Second section is defining the problem formulation. After that the mathematical modeling and its solution methodology are proposed in section 3 and 4, respectively. Finally, some the conclusion of this study is presented with some suggestions for further research in this research area.

† :Corresponding Author

2. PROBLEM FORMULATION

An n items inventory problem is considered here, where the demand of each items are constant and deterministic. The demand rate of item i is D_i units per unit time. Replenishment of item i is instantaneous with size of y_i units and unit-purchasing price of $\$c_i$. A joint replenishment is conducted periodically every T time intervals. However, all items may not be included in each replenishment. Item i is only included every Z_iT time intervals. In other word, Z_iT is the cycle time of item i. A replenishment incurs of major ordering cost $\$K$ and minor ordering cost $\$k_i$ for every item i included in the replenishment. Each lot received contains p_i fraction of defectives, in which $0 < p_i < 1$, with a known probability density function $f_i(p_i)$. For identifying defectives, a 100% screening process of the lot is conducted at a rate of x_i where $x_i > D_i$. The screening cost of item i is $\$d_i$ per unit. Items of poor quality are sorted, kept in stock and sold at a salvage value of $\$v_i$ per unit as a single batch within the replenishment cycle. The inventory holding cost of item i is $\$h_i$ per unit per unit time and the selling price of good-quality items is $\$s_i$ per unit. Shortages are allowed and completely backordered. The maximum backordering quantity of item i is B_i and the backordering cost of item i is $\$b_i$ per unit. The decision to be taken in this situation is determining the basic time cycle T, replenishment multiplier Z_i, and backordering quantity B_i in order to maximize the expected total profit per unit time.

Problem Assumption

1. The demand rate for each item is constant and deterministic.
2. The replenishment lead time is known and constant.
3. The replenishment is instantaneous.
4. The entire order quantity is delivered at the same time.
5. The screening process and demand for all items proceed simultaneously, but the screening rates are greater than demand rates, $x_i > D_i$.
6. The defective items exist in lot size y_i. The defective percentage, p_i, has a uniform distribution with $[\alpha_i, \beta_i]$, where $0 < \alpha_i < \beta_i < 1$.
7. Shortage is completely backordered.

3. MATHEMATICAL MODEL

The inventory cycle for item i with imperfect quality and complete backordering is illustrated in Figure 1. After screening process is completed at time t, the inventory level is reduced by p_iy_i unit due to the withdrawal of defective items. Stock shortage are allowed and completely backlogged at the beginning of each cycle. Shortages are avoided within the screening time, hence p_i is restricted to

$$p_i \le 1 - \frac{D_i}{x_i} \tag{1}$$

Total revenue per cycle of item i can be calculated as

$$TR_i = (1 - p_i)\, y_i s_i + p_i y_i v_i \tag{2}$$

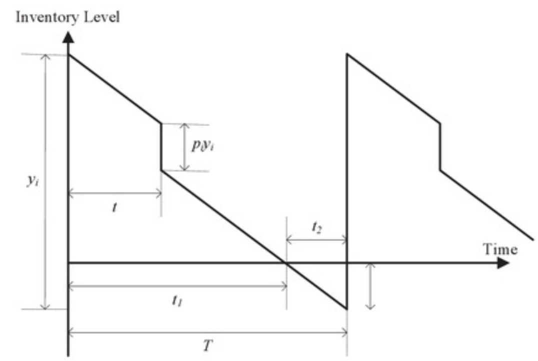

Figure 1. Inventory system behavior

Since $Z_iT = (1-p_i)\, y_i / D_i$, the relationship between order size and replenishment time can be stated as

$$y_i = \frac{Z_iTD_i}{(1-p_i)} \tag{3}$$

Hence, after substituting Eq. (3) into Eq. (2) and some algebra, the total revenue per unit time is

$$TRU = \sum_{i=1}^{n} D_i \left(s_i + \frac{p_i}{(1-p_i)} v_i \right) \tag{4}$$

Therefore, the expected total revenue per unit time for uniformly distributed defective can be formulated as

$$ETRU = \sum_{i=1}^{n} D_i \left(s_i + \frac{\alpha_i + \beta_i}{2} \frac{1}{\beta_i - \alpha_i} \ln\left(\frac{1-\alpha_i}{1-\beta_i} \right) v_i \right) \tag{5}$$

The sum of purchasing cost, screening cost, holding cost, and backordering cost of item i can be stated as

$$TC_i = c_i y_i + d_i y_i + h_i \left(\frac{1}{2} \frac{(y_i - p_i y_i - B_i)^2}{D_i} + \frac{p_i {y_i}^2}{x_i} \right) + \frac{1}{2} \frac{b_i {B_i}^2}{D_i} \tag{6}$$

Adding the major and minor ordering cost for the case of joint replenishment, the total cost per unit time can be calculated as

$$TCU = \frac{K}{T} + \frac{\sum_{i=1}^{n} \frac{k_i}{Z_i}}{T} + \sum_{i=1}^{n} \frac{TC_i}{Z_i T} \qquad (7)$$

After some algebra and substituting the probability density function of the percentage defective items, the expectation of total cost can be formulated as

$$\begin{aligned} ETCU = {} & \frac{K}{T} + \frac{\sum_{i=1}^{n} \frac{k_i}{Z_i}}{T} + \sum_{i=1}^{n} (c_i + d_i) D_i \frac{1}{\beta_i - \alpha_i} \ln\left(\frac{1-\alpha_i}{1-\beta_i}\right) \\ & + h_i \left(\frac{1}{2} T Z_i D_i - B_i + \frac{1}{2} \frac{B_i^2}{Z_i T D_i} + \right. \\ & \left. \frac{T Z_i D_i^2}{x_i} \frac{\beta_i + \alpha_i}{2} \frac{1}{(1-\alpha_i)(1-\beta_i)} \right) + \frac{1}{2} \frac{b_i B_i^2}{D_i T Z_i} \end{aligned} \qquad (8)$$

The expected net profit per unit time, ETPU, is determined by the expected total revenue per unit time less the expected total cost per unit time

$$ETPU = ETRU - ETCU \qquad (9)$$

Therefore, the expected net profit per unit time can be stated as function of variables, i.e. $ETPU = f(T, Z_i, B_i)$

4. SOLUTION METHODOLOGY

The expression of *ETPU* is consist of both real variables, i.e. T and B_i, and integer variables, i.e. Z_i. Therefore, an iterative procedure is proposed here:

Step 1. Set initial value of $Z_i = 1$, initial value of $B_i = 0$.
Step 2. Given Z_i and B_i, find the optimal value of T. This is a single variable optimization of T, see section 4.1 for the derivation of T.
Step 3. Given T and B_i, find the optimal value of Z_i. The condition for obtaining Z_i is presented in section 4.2
Step 4. Given T and Z_i, the optimal value of B_i can be found independently, see section 4.3 for the derivation.
Step 5. Repeat from step 2 until convergence is reached, i.e. the current result of T is similar with previous calculated T.

4.1. Derivation of T in the Step 2

Given the value of Z_i and B_i, the *ETPU* is function of single variable T. Set the necessary condition of single variable optimization, i.e. the first derivative of the function equal to zero, one can obtain

$$T^* = \sqrt{\frac{\left(K + \sum_{i=1}^{n}\left(\frac{k_i}{Z_i} + \frac{h_i}{2}\frac{B_i^2}{Z_i D_i} + \frac{1}{2}\frac{b_i B_i^2}{D_i Z_i}\right)\right)}{\sum_{i=1}^{n}\left(\frac{h_i}{2} Z_i D_i + \frac{h_i Z_i D_i^2}{x_i}\frac{\beta_i + \alpha_i}{2(1-\alpha_i)(1-\beta_i)}\right)}} \qquad (10)$$

For the first iteration, where all $Z_i = 1$ and $B_i = 0$, the optimal value of T can be simplified into

$$T^* = \sqrt{\frac{\left(K + \sum_{i=1}^{n} k_i\right)}{\sum_{i=1}^{n} h_i \left(\frac{1}{2} D_i + \frac{D_i^2}{x_i}\frac{\beta_i + \alpha_i}{2(1-\alpha_i)(1-\beta_i)}\right)}} \qquad (11)$$

It is noted that the second derivative of *ETPU* with respect to T can be derived as

$$\frac{d^2 ETPU}{dT^2} = -\frac{2}{T^3}\left(K + \sum_{i=1}^{n}\left(\frac{k_i}{Z_i} + \frac{h_i}{2}\frac{B_i^2}{Z_i D_i} + \frac{1}{2}\frac{b_i B_i^2}{D_i Z_i}\right)\right) \qquad (12)$$

Therefore, it can be shown that T^* calculated in Equation (10) or (11) is a maximum point.

4.2. Obtaining Z_i in the Step 3

Given any value of B_i and T, the *ETPU* is a multi variable optimization of integer Z_i. The expression of *ETPU* can be rewritten as follow

$$ETPU = ETPU_0 - \sum_{i=1}^{n} ETPU_i(Z_i) \qquad (13)$$

Therefore, the problem of maximizing *ETPU* can be dividing into problems of minimizing $ETPU_i(Z_i)$ independently. Since Z_i is a discrete integer, the optimality conditions of $ETPU_i(Z_i)$ are

$$ETPU_i(Z_i^*) \le ETPU_i(Z_i^* + 1) \qquad (14)$$

and

$$ETPU_i(Z_i^*) \le ETPU_i(Z_i^* - 1) \qquad (15)$$

After some algebra, the optimality condition for Z_i can be written as follow

$$Z_i^*(Z_i^* - 1) \le \Psi_i \le Z_i^*(Z_i^* + 1) \qquad (16)$$

where

$$\Psi_i = \frac{1}{T^2} \frac{k_i + \frac{B_i^2}{2D_i}(h_i + b_i)}{\frac{h_i}{2} D_i + \frac{h_i D_i^2}{x_i}\frac{\beta_i + \alpha_i}{2(1-\alpha_i)(1-\beta_i)}} \qquad (17)$$

4.3. Derivation of B_i in the Step 4

Given any value of Z_i and T, the $ETPU$ is a multi variable optimization of B_i. Since B_i are real variable, one can take the first derivative of the function equal to zero as the necessary optimality condition.

$$\frac{\partial}{\partial B_i} ETPU = h_i - \frac{B_i}{TZ_iD_i}(h_i + b_i) = 0 \tag{18}$$

Therefore, the optimal value of B_i can be stated as

$$B_i{}^* = \frac{h_i}{(h_i + b_i)} Z_iTD_i \tag{19}$$

It is noted that the second partial derivative of B_i are

$$\frac{\partial^2}{\partial B_i{}^2} ETPU = -\frac{(h_i + b_i)}{TZ_iD_i} \quad \forall i = 1\ldots n \tag{20}$$

$$\frac{\partial^2}{\partial B_i \partial B_j} ETPU = 0 \quad \forall i \neq j \tag{21}$$

Hence, one can prove that the Hessian matrix is negative definite. Therefore, it can be shown that the $B_i{}^*$ is the maximum point.

5. NUMERICAL EXAMPLE

In order to show the applicability of the proposed method, a numerical example of 4 items inventory problem is presented here with. The major ordering cost is $40. The other problem parameters are presented in the Table 1.

Table 1: Problem Parameters

i	1	2	3	4
D_i	10000	2000	4000	500
c_i	20	5	30	15
h_i	2	0.5	3	1.5
x_i	20000	3000	7500	2500
d_i	0.4	0.2	0.3	0.5
v_i	10	2.5	15	7.5
k_i	15	15	15	15
α_i	0	0	0	0
β_i	0.25	0.3	0.2	0.25
b_i	5	3	4	10

Following proposed methodology in Section 4, the value of T, Z_i, and B_i can be calculated as shown in Table 2. It is noted that the process is convergence after 6 iterations.

Table 2: Calculation Steps

Iteration	T	i	1	2	3	4
1	0.0716	Z_i	1	1	1	1
		B_i	0	0	0	0
2	0.0665	Z_i	1	2	1	3
		B_i	204.55	40.91	122.73	14.01
3	0.0660	Z_i	1	2	1	3
		B_i	190.05	38.01	114.03	13.01
4	0.0660	Z_i	1	2	1	3
		B_i	188.63	37.73	113.18	12.92
5	0.0660	Z_i	1	2	1	3
		B_i	188.50	37.70	113.10	12.91
6	0.0660	Z_i	1	2	1	3
		B_i	188.48	37.70	113.09	12.91
7	0.0660	Z_i	1	2	1	3
		B_i	188.48	37.70	113.09	12.91

6. CONCLUDING REMARK

This paper is successfully presented a mathematical model for joint replenishment inventory system considering imperfect quality items and shortage. A solution methodology for solving the model is also proposed altogether with one numerical example. This model and solution methodology is ready to be extended to other problem variants that are commonly exist in the inventory research area.

ACKNOWLEDGMENT

This work is partially supported by Directorate General of Higher Education, Ministry of Education and Culture, Republic Indonesia under International Research Collaboration and Scientific Publication Research Grant No. 086/SP2H/PL/DIT.LITABMAS/V/2013, No. 1317/K5/KM/2014, and No. 005/HB-LIT/III/2015.

REFERENCES

Chang, H. C., & Ho, C. H. (2010). Exact closed-form solutions for "optimal inventory model for items with imperfect quality and shortage backordering". *Omega*, *38*(3), 233-237.

Khouja, M., & Goyal, S. (2008). A review of the joint replenishment problem literature: 1989–2005. *European Journal of Operational Research*, *186*(1), 1-16.

Konstantaras, I., Goyal, S. K., & Papachristos, S. (2007). Economic ordering policy for an item with imperfect quality subject to the in-house inspection. *International Journal of Systems Science*, *38*(6), 473-482.

Moon, I. K., Goyal, S. K., & Cha, B. C. (2008). The joint replenishment problem involving multiple suppliers offering quantity discounts. *International Journal of Systems Science*, *39*(6), 629-637.

Paul, S., Wahab, M.I.M., & Ongkunaruk, P. (2014). Joint replenishment with imperfect items and price discount. *Computers and Industrial Engineering*, 74 (1), 179-185.

Rezaei, J. (2005). Economic order quantity model with backorder for imperfect quality items. In *Proceedings of IEEE International Engineering Management Conference* (pp. 466-470).

Salameh, M. K., & Jaber, M. Y. (2000). Economic production quantity model for items with imperfect quality. *International Journal of Production Economics*, *64*(1), 59-64.

Tsai, C. Y., Tsai, C. Y., & Huang, P. W. (2009). An association clustering algorithm for can-order policies in the joint replenishment problem. *International Journal of Production Economics*, *117*(1), 30-41.

Wang, L., He, J., Wu, D., & Zeng, Y. R. (2012). A novel differential evolution algorithm for joint replenishment problem under interdependence and its application. *International Journal of Production Economics*, *135*(1), 190-198.

Wee, H. M., Yu, J., & Chen, M. C. (2007). Optimal inventory model for items with imperfect quality and shortage backordering. *Omega*, *35*(1), 7-11.

Yoo, S. H., Kim, D., & Park, M. S. (2009). Economic production quantity model with imperfect-quality items, two-way imperfect inspection and sales return. *International Journal of Production Economics*, *121*(1), 255-265.

Paper 33

ORIGINALITY REPORT

33%	21%	28%	15%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	Wee, H.M.. "Optimal inventory model for items with imperfect quality and shortage backordering", Omega, 200702 Publication	4%
2	B. C. Cha, I. K. Moon. "The joint replenishment problem with quantity discounts under constant demand", OR Spectrum, 2005 Publication	3%
3	www.science.gov Internet Source	3%
4	www.growingscience.com Internet Source	2%
5	www.tandfonline.com Internet Source	2%
6	Filemon Yoga Adhisatya, The Jin Ai, Dah-Chuan Gong. "Economic Lot Scheduling Problem with two Imperfect Key Modules", Procedia Manufacturing, 2015 Publication	2%

7	link.springer.com Internet Source	1%
8	Submitted to University of Johannsburg Student Paper	1%
9	china.iopscience.iop.org Internet Source	1%
10	Submitted to Universitas Atma Jaya Yogyakarta Student Paper	1%
11	Gong, Dah-Chuan, Jia-Lun Kang, Gary C. Lin, and T. C. Hou. "On an IC wire bonding machine production–inventory problem with time value of money", International Journal of Production Research, 2016. Publication	1%
12	Khan, M.. "A review of the extensions of a modified EOQ model for imperfect quality items", International Journal of Production Economics, 201107 Publication	1%
13	Submitted to Indian Institute of Technology, Kharagpure Student Paper	1%
14	mafiadoc.com Internet Source	1%
15	Submitted to International University - VNUHCM	

Student Paper

1%

16	www.rairo-ro.org Internet Source	<1%
17	Y. Chen, M.I.M. Wahab, P. Ongkunaruk. "A joint replenishment problem considering multiple trucks with shipment and resource constraints", Computers & Operations Research, 2016 Publication	<1%
18	yujor.fon.rs Internet Source	<1%
19	Submitted to Lebanese American University Student Paper	<1%
20	Submitted to Northcentral Student Paper	<1%
21	Proceedings of the Institute of Industrial Engineers Asian Conference 2013, 2013. Publication	<1%
22	iasir.net Internet Source	<1%
23	www.ijmems.in Internet Source	<1%
24	Lin Wang, Cai-Xia Dun, Wen-Jie Bi, Yu-Rong Zeng. "An effective and efficient differential evolution algorithm for the integrated stochastic	<1%

joint replenishment and delivery model", Knowledge-Based Systems, 2012
Publication

25	S G Johansen, P Melchiors. "Can-order policy for the periodic-review joint replenishment problem", Journal of the Operational Research Society, 2017 Publication	<1 %
26	Yanru Chen, Lu Yang, Yangsheng Jiang, M.I.M. Wahab, Jie Yang. "Joint replenishment decision considering shortages, partial demand substitution, and defective items", Computers & Industrial Engineering, 2018 Publication	<1 %
27	www.emeraldinsight.com Internet Source	<1 %
28	Ali, S.M.. "Analytical thermal hydraulic model for oscillatory condensation of steam in presence of air", Nuclear Engineering and Design, 200710 Publication	<1 %
29	www.scribd.com Internet Source	<1 %
30	Guchhait, Partha, Manas Kumar Maiti, and Manoranjan Maiti. "Production-inventory models for a damageable item with variable demands and inventory costs in an imperfect production	<1 %

process", International Journal of Production Economics, 2013.
Publication

31 freidok.uni-freiburg.de <1%
Internet Source

32 Prerna Gautam, Aakanksha Kishore, Aditi Khanna, Chandra K. Jaggi. "Strategic defect management for a sustainable green supply chain", Journal of Cleaner Production, 2019 <1%
Publication

33 Jia-Tzer Hsu, Lie-Fern Hsu. "An integrated vendor–buyer inventory model with imperfect items and planned back orders", The International Journal of Advanced Manufacturing Technology, 2013 <1%
Publication

34 S. Paul, M.I.M. Wahab, P. Ongkunaruk. "Joint replenishment with imperfect items and price discount", Computers & Industrial Engineering, 2014 <1%
Publication

35 Ligang Cui, Lin Wang, Jie Deng, Jinlong Zhang. "Intelligent algorithms for a new joint replenishment and synthetical delivery problem in a warehouse centralized supply chain", Knowledge-Based Systems, 2015 <1%
Publication

36	Teng, Hui Ming, and Ping Hui Hsu. "Revisit economic ordering strategy for items with imperfect quality and backordering", International Journal of Management and Enterprise Development, 2013. Publication	<1%
37	Qing Li. "Solving the multi-buyer joint replenishment problem with the RAND method", Computers & Industrial Engineering, 2004 Publication	<1%
38	Hsu, Lie-Fern, and Jia-Tzer Hsu. "Economic production quantity (EPQ) models under an imperfect production process with shortages backordered", International Journal of Systems Science, 2014. Publication	<1%

Exclude quotes	Off	Exclude matches	Off
Exclude bibliography	On		