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by The Jin Ai

Three Approaches to Find Optimal Production Run Time of an Imperfect Production System

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Abstract This paper considers an Economic Production Quantity (EPQ) model where a product is to be manufactured in batches on an imperfect production system over infinite planning horizon. During a production run of the product, the production system is dictated by two unreliable key production subsystems (KPS) that may shift from an in-control to an out-of-control state due to three independent sources of shocks. A mathematical model describing this situation has been developed by Lin and Gong (2011) in order to determine production run time that minimizes the expected total cost per unit time including setup, inventory carrying, and defective costs. Since the optimal solution with exact closed form of the model cannot be obtained easily, this paper considered three approaches of finding a near-optimal solution. The first approach is using Maclaurin series to approximate any exponential function in the objective function and then ignoring cubic terms found in the equation. The second approach is similar with first approach but considering all terms found. The third approach is using Golden Section search directly on the objective function. These three approaches are then compared in term computational efficiency and solution quality of through some numerical experiments.

Keywords EPQ model · Imperfect production system · Optimization technique · Approximation and numerical method

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1 Introduction

The problem considered in this paper had been formulated by Lin and Gong (2011) as follow. A product is to be manufactured in batches on an imperfect production system over an infinite planning horizon. The demand rate is d, and the production rate is p. The imperfectness of the system is shown on two imperfect key production subsystems (KPS) that may shift from an in-control to an out-of-control state due to three independent sources of shocks: source 1's shock causes first KPS to shift, source 2's shock causes second KPS to shift, and source 3's shock causes both KPS to shift. Each shock occurs at random time U_1, U_2, and U_{12} that follows exponential distribution with mean $1/\lambda_1$, $1/\lambda_2$, and $1/\lambda_{12}$, respectively. When at least one KPS on out-of-control state, consequently, the production system will produced some defective items with fixed but different rates: α percentage when first KPS out-of-control, β percentage when the second KPS out-of-control, and δ percentage when the both KPS out-of-control. The cost incurred by producing defective items when the first KPS is shifted, the second KPS is shifted, and both KPS are shifted are π_1, π_2, and π_{12}, respectively. The optimization problem is to determining optimal production run time τ that minimizes the expected total cost per unit time including setup, inventory carrying, and defective costs. It is noted that the unit setup cost is A and inventory carrying per unit per unit time is h.

As derived in Lin and Gong (2011), the objective function of this optimization model is given by following equations.

$$Z(\tau) = \frac{Ad}{p\tau} + \frac{h(p-d)\tau}{2} + \frac{d(\pi_1 E[N_1(\tau)] + \pi_2 E[N_2(\tau)] + \pi_{12} E[N_{12}(\tau)])}{p\tau} \tag{1}$$

$$E[N_1(\tau)] = p\alpha\left(\frac{1-\exp[-(\lambda_2+\lambda_{12})\tau]}{\lambda_2+\lambda_{12}} - \frac{1-\exp[-(\lambda_1+\lambda_2+\lambda_{12})\tau]}{\lambda_1+\lambda_2+\lambda_{12}}\right) \tag{2}$$

$$E[N_2(\tau)] = p\beta\left(\frac{1-\exp[-(\lambda_1+\lambda_{12})\tau]}{\lambda_1+\lambda_{12}} - \frac{1-\exp[-(\lambda_1+\lambda_2+\lambda_{12})\tau]}{\lambda_1+\lambda_2+\lambda_{12}}\right) \tag{3}$$

$$\begin{aligned} E[N_{12}(\tau)] = p\delta\Bigg(& \frac{\exp[-(\lambda_1+\lambda_{12})\tau] + (\lambda_1+\lambda_{12})\tau - 1}{\lambda_1+\lambda_{12}} \\ & + \frac{\exp[-(\lambda_2+\lambda_{12})\tau] + (\lambda_2+\lambda_{12})\tau - 1}{\lambda_2+\lambda_{12}} \\ & - \frac{\exp[-(\lambda_1+\lambda_2+\lambda_{12})\tau] + (\lambda_1+\lambda_2+\lambda_{12})\tau - 1}{\lambda_1+\lambda_2+\lambda_{12}}\Bigg) \end{aligned} \tag{4}$$

Although the problem is a single variable optimization, the optimal solution with exact closed form of the model cannot be obtained easily. Therefore, this paper considered three approaches of finding a near-optimal solution. These approaches are then compared through some numerical experiments.

2 Approaches of Finding a Near-Optimal Solution

2.1 First Approach

In this first approach, following Maclaurin series is applied to approximate any exponential function in the objective function.

$$\exp(-\lambda\tau) \approx 1 - \lambda\tau + \frac{1}{2!}(\lambda\tau)^2 - \frac{1}{3!}(\lambda\tau)^3 \tag{5}$$

Therefore after some algebra, the objective function can be approximated as following equations.

$$Z(\tau) \approx Z_1(\tau) = \frac{Ad}{p\tau} + \frac{H\tau}{2} - \frac{B\tau^2}{6} \tag{6}$$

$$H = h(p-d) + d(\pi_1\alpha\lambda_1 + \pi_2\beta\lambda_2 + \pi_{12}\delta\lambda_{12}) \tag{7}$$

$$B = d[\pi_1\alpha\lambda_1(\lambda_1 + 2\lambda_2 + 2\lambda_{12}) + \pi_2\beta\lambda_2(2\lambda_1 + \lambda_2 + 2\lambda_{12}) - \pi_{12}\delta(2\lambda_1\lambda_2 - \lambda_{12}^2)] \tag{8}$$

From calculus optimization, it is known that the necessary condition for obtaining the minimum value of Z_1 is set the first derivative equal to zero. Applying this condition for Eq. (6), it is found that

$$\frac{dZ_1(\tau)}{d\tau} = -\frac{Ad}{p\tau^2} + \frac{H}{2} - \frac{2B\tau}{6} = 0 \tag{9}$$

Equation (9) can be rewritten as

$$2B\tau^3 - 3H\tau^2 + 6Ad/p = 0. \tag{10}$$

If the cubic term in Eq. (10) is ignored, then a near-optimal solution of the first approach can be obtained as follow

$$\tau_1^* = \sqrt{\frac{2Ad}{p[h(p-d) + d(\pi_1\alpha\lambda_1 + \pi_2\beta\lambda_2 + \pi_{12}\delta\lambda_{12})]}} \tag{11}$$

2.2 Second Approach

The second approach is developed based on Eq. (9). Another near-optimal solution can be found as the root of this equation. Bisection algorithm can be applied here to find the root of this equation (τ_2*) with lower searching bound of $\tau_L = 0$ and upper searching bound

$$\tau_U = \sqrt{\frac{2Ad}{ph(p-d)}} \tag{12}$$

It is noted that the lower bound is selected equal to zero due to the fact that the optimal production run time have to be greater than zero. While the upper bound is selected as Eq. (12) due to the fact that the optimal production run time in the presence of imperfectness, i.e. with non negative values of α, β, and δ, is always smaller than the optimal production run time of perfect production system, i.e. with zero values of α, β, and δ. Substituting $\alpha = \beta = \delta = 0$ to Eq. (11) provides the same value as optimal production run time of classical and perfect EPQ (Silver et al. 1998), as shown in the right hand side of Eq. (12). The detail of bisection algorithm can be found in any numerical method textbook, i.e. Chapra and Canale (2002).

2.3 Third Approach

The third approach is using pure numerical method to find the minimum value of Z based on Eq. (1). The Golden Section method is applied here using the same bound as the second approach. Therefore, the searching of the optimal production run time of this approach (τ_3*) is conducted at interval $\tau_L \le \tau_3^* \le \tau_U$, where $\tau_L = 0$ and τ_U is determined using Eq. (12). Further details on the Golden Section method can be found in any optimization textbook, i.e. Onwubiko (2000).

3 Numerical Experiments

Numerical experiments are conducted in order to test the proposed approaches for finding the optimal production run time. Nine problems (P1, P2, …, P9) are defined for the experiments and the parameters of each problem are presented in Table 1. The result of all approaches are presented in Table 2, which comprise of the optimal production run time calculated from each approach (τ_1*, τ_2*, τ_3*) and their corresponding expected total cost [$Z(\tau_1*)$, $Z(\tau_2*)$, $Z(\tau_3*)$]. Some metrics defined below are also presented in Table 2 in order to compare the proposed approaches.

In order to compare the proposed approaches, since the optimal expected total cost cannot be exactly calculated, the best expected total cost is defined as following equation

$$Z^* = \min\{Z(\tau_1^*), Z(\tau_2^*), Z(\tau_3^*)\} \tag{13}$$

Table 1 Problem parameters of the numerical experiments

Parameters	P1	P2	P3	P4	P5	P6	P7	P8	P9
p	300	300	300	300	300	300	300	300	300
d	200	200	200	200	200	200	200	200	200
A	100	100	100	100	100	100	100	100	100
h	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
π_1	10	10	10	10	10	10	10	10	10
π_2	10	10	10	10	10	10	10	10	10
π_{12}	12	12	12	12	12	12	12	12	12
α	0.1	0.1	0.1	0.3	0.3	0.3	0.5	0.5	0.5
β	0.1	0.1	0.1	0.3	0.3	0.3	0.5	0.5	0.5
δ	0.16	0.16	0.16	0.48	0.48	0.48	0.8	0.8	0.8
λ_1	0.05	0.15	0.25	0.05	0.15	0.25	0.05	0.15	0.25
λ_2	0.1	0.3	0.5	0.1	0.3	0.5	0.1	0.3	0.5
λ_{12}	0.02	0.06	0.1	0.02	0.06	0.1	0.02	0.06	0.1

If τ^* is the best value of production run time in which $Z^* = Z\,(\tau^*)$, the deviation of the solution of each approach from the best solution is calculated from following equation

$$\Delta\tau_i = \frac{|\tau_i{}^* - \tau^*|}{\tau^*} \times 100\,\% \tag{14}$$

Furthermore, the deviation of the expected total cost of each approach from the best one can be determined using following equation

$$\Delta Z_i = \frac{Z(\tau_i^*) - Z^*}{Z^*} \times 100\,\% \tag{15}$$

Regarding to solution quality, it is shown in Table 2 that the third approach is consistently providing the best expected total cost across nine test problems among

Table 2 Results of the numerical experiments

Result	P1	P2	P3	P4	P5	P6	P7	P8	P9
$\tau_1{}^*$	1.7085	1.0496	0.8239	1.0496	0.6198	0.4823	0.8239	0.4823	0.3746
$\tau_2{}^*$	1.8067	1.2035	1.0199	1.0897	0.6679	0.5350	0.8489	0.5104	0.4046
$\tau_3{}^*$	1.7986	1.1750	0.9642	1.0877	0.6633	0.5280	0.8479	0.5084	0.4016
$\Delta\tau_1$	5.01 %	10.67 %	14.55 %	3.51 %	6.56 %	8.65 %	2.83 %	5.14 %	6.72 %
$\Delta\tau_2$	0.46 %	2.43 %	5.78 %	0.18 %	0.70 %	1.33 %	0.11 %	0.40 %	0.72 %
$\Delta\tau_3$	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %
$Z(\tau_1{}^*)$	76.18	120.96	151.69	124.89	208.55	265.54	159.60	269.73	344.77
$Z(\tau_2{}^*)$	76.09	120.34	150.38	124.82	208.12	264.61	159.54	269.39	344.02
$Z(\tau_3{}^*)$	76.09	120.31	150.20	124.82	208.12	264.59	159.54	269.38	344.02
ΔZ_1	0.12 %	0.54 %	0.99 %	0.06 %	0.21 %	0.36 %	0.04 %	0.13 %	0.22 %
ΔZ_2	0.00 %	0.02 %	0.12 %	0.00 %	0.00 %	0.01 %	0.00 %	0.00 %	0.00 %
ΔZ_3	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %

three approaches. It is also shown in the Table 2 that $Z(\tau_1^*) > Z(\tau_2^*) > Z(\tau_3^*)$, while the deviations of the expected total cost of the first and second approaches are less than 1.0 and 0.2 %, respectively. Furthermore, it is found that production run time found by the three approaches are $\tau_1^* < \tau^* = \tau_3^* < \tau_2^*$. The deviations of the first approach solution from the best solution are less than 14.6 %, while the deviations of the second approach solution from the best solution are less than 5.8 %.

These results show that the first approach is able to find reasonable quality solution of the problems although its computational effort is very simple compare to other approaches. It is also implied from these result that the Maclaurin approximation used in the second approach is effective to support the second approach finding very close to best solution of the problems, although the computational effort of the second approach is higher than the computational effort of the first approach. Since the third approach is using the highest computational effort among the proposed approaches, it can provide the best solution of the problems.

4 Concluding Remarks

This paper proposed three approaches for solving Lin and Gong (2011) model on Economic Production Quantity in an imperfect production system. The first approach is incorporating Maclaurin series and ignoring cubic terms in the first derivative of total cost function. The second approach is similar with the first approach but incorporating all terms in the total cost function, then using Bisection algorithm for finding the root of the first derivative function. The third approach is using Golden Section method to directly optimize the total cost function. Numerical experiments show that the third approach is able to find the best expected total cost among the proposed approaches but using the highest computational effort. It is also shown that the first approach is able to find reasonable quality solution of the problems despite the simplicity of its computational effort.

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