

# JURNAL TEKNOLOGI INDUSTRI

*High Order Discontinuous Galerkin for  
Numerical Simulation of Elastic Wave Propagation*

Sistem Perangkat Lunak Berbasis Web  
untuk Sarana Kolaborasi Desain

Perencanaan Produksi Berhirarki Produk Olahan Kayu  
Menggunakan Model *Goal Programming*

Penentuan Faktor-faktor yang Berpengaruh  
terhadap Karakteristik Kualitas Tebal Plastik  
dengan Metode Taguchi

*Watermarking Citra Warna Digital Menggunakan Alihragam  
Wavelet Daubechies dan Strategi Penyisipan Watermark  
pada Subbidang Detail Citra*

Peningkatan Efisiensi Algoritma Simpleks:  
Modifikasi dengan Metoda Kenaikan Terbesar

Implementasi Algoritma Welch-Powell  
dalam Pola Perancangan Lampu Lalu Lintas

Pemodelan Dinamis Linier dalam Sistem Produksi

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*High Order Discontinuous Galerkin for Numerical Simulation of Elastic Wave Propagation*

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## High Order Discontinuous Galerkin for Numerical Simulation of Elastic Wave Propagation

In which  $(u, v)$  is the velocity vector,  $\sigma$  is the stress tensor,  $(f, g)$  is the body force,  $\Gamma$  is the boundary,  $\nu$  is the normal unit vector, the method only requires communication between elements that have common faces. No global matrix inversion is needed and the solution is close to that of finite element method. The DG method can be regarded as closer to the weak form than the finite difference method. The numerical solution is obtained by solving a system of linear equations.

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### Abstract

We present a study of elastic wave propagation in isotropic media. The Discontinuous Galerkin Method is applied to solve the elastodynamic equations which represent elastic wave propagation. The elastodynamic equations are transformed into a stress-velocity formulation. The Discontinuous Galerkin Method is a finite element that allows a discontinuity of the numerical solution at element interface. Through a proper choice of the flux computation points, the method only requires communication between elements that have common faces. The utilization of high-order Legendre polynomials as basis functions has been shown to be more efficient in reducing the numerical dispersion and numerical dissipation. Discontinuous Galerkin Method is a compact method, high-order basis functions can be used easily without any essentially difficulty and even spectral accuracy becomes obtainable. Temporal discretization utilized explicit staggered leapfrog method. We compare the numerical results to the exact solutions and the comparison shows a good agreement.

**Keywords:** elastic wave propagation, discontinuous Galerkin, high order basis functions

### 1. Introduction

The discontinuous galerkin (DG) method originally developed by Reed and Hill for the solution of neutron transport problem. Le Saint and Raviart were the first to put the method on a firm mathematical base Cockburn and Shu (1997; 1998). The technique lay dormant for several years before becoming popular. Cockburn and Shu (1997; 1998) developed DG method to solve convection-diffusion problem and extended it to multidimensional systems case. Recently, the DG formulation of the finite element method has been increasingly use in computational fluid dynamics (CFD) (Karniadakis & Sherwin, 1999; Van Der Vegt & Van der Ven, 1998), computational electromagnetic (CEM) (Hesthaven & Warburton, 2001) and computational aeroacoustics (CAA) (Atkins & shu, 1996; Atkins, 1997).

It is well known that highly accurate methods are required for long time simulations of wave propagation, which are essentially non dispersive and non dissipation (Hu & Rasetarinera, 1999). Many numerical schemes such as finite difference and finite element are developed to study wave propagation. Among the schemes, the DG method provides an attractive approach to solve problems containing discontinuities, such as those arise in hyperbolic systems. The DG method allows more general mesh configuration and interelement continuity is not required. The basis function is discontinuous across mesh boundaries. Through a proper choices of flux

computation points, the method only requires communication between mesh that have common faces. No global matrix inversion is required and the problem can be solved locally in each mesh. The DG method can be regarded as cross between a finite volume and finite element method and it has many of the good properties of both.

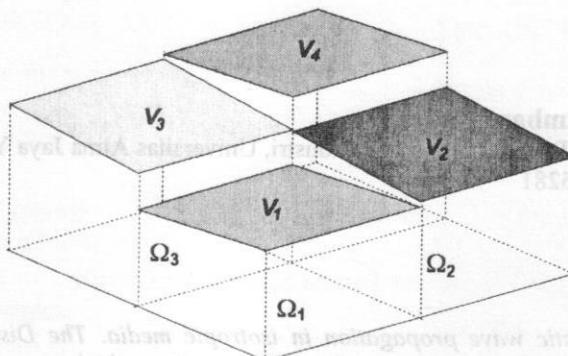


Figure 1. Mesh and Solution of Discontinuous Galerkin Method

Lowrie (1996) considered the space-time DG method which involves discontinuous mesh in both space and time to solve hyperbolic laws (Lowrie, 1996). The method requires excessive resources to be useful for practical application. Atkins and Shu (1996) described a quadrature-free formulaton for DG method. They apply the formulation to linearized Euler equations. Stanescu *et al.* combined DG method with spectral element method to solve aircraft engine noise scattering (Stanescu *et al.*). They used high-order Legendre polynomials as basis functions. The numerical results show that trends of the noise field are well predicted. Li (1996) developed adaptive space-time DG method to solve elastodynamic equations. Triangular mesh and linear basis function were used. The numerical results have a good agreement with exact solution.

In this paper, elastodynamic equations, which described elastic wave propagation, are solved using DG method. High order Legendre polynomials are used as basis functions. The elastodynamic equations will be discretized using rectangular mesh and explicit Leapfrog method is used as time integration method.

## 2. Governing Equations

The Elastodynamic Equations are set of linear hyperbolic equations. The equations are transformed into the following first-order hyperbolic system via stress-velocity formulation.

$$\frac{\partial v_x}{\partial t} = \frac{1}{\rho} \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \right) + f_x \quad (1.a)$$

$$\frac{\partial v_y}{\partial t} = \frac{1}{\rho} \left( \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \right) + f_y \quad (1.b)$$

$$\frac{\partial \tau_{xx}}{\partial t} = (\lambda + 2\mu) \frac{\partial v_x}{\partial x} + \mu \frac{\partial v_y}{\partial y} \quad (1.c)$$

$$\frac{\partial \tau_{yy}}{\partial t} = (\lambda + 2\mu) \frac{\partial v_y}{\partial y} + \mu \frac{\partial v_x}{\partial x} \quad (1.d)$$

$$\frac{\partial \tau_{xz}}{\partial t} = \mu \left( \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right) \quad (1.e)$$

In which  $(v_x, v_y)$  is the velocity vector,  $(\tau_{xx}, \tau_{yy}, \tau_{xy})$  is the stress tensor,  $(f_x, f_y)$  is the body force vector,  $\rho$  is the density,  $\lambda$  and  $\mu$  are Lame coefficient. Stress-velocity formulation for one-dimensional propagation in the  $x$  direction are:

$$\frac{\partial v_y}{\partial t} - \frac{1}{\rho} \frac{\partial \tau_{xy}}{\partial x} = 0 \quad (2.a)$$

$$\frac{\partial \tau_{xy}}{\partial t} - \mu \frac{\partial v_y}{\partial x} = 0 \quad (2.b)$$

The numerical flux  $(\tau_{xy})$  at the interface between elements is approximated by using average

### 3. DG Discretization

#### a. One Dimensional Discretization

We present one-dimensional DG Discretization in one dimension to begin with, and extend them to two dimensions. We divide the interval of domain into subintervals  $\Omega_j = [x_j, x_{j+1}]$ ,  $j = 0, \dots, N$ . In each subinterval (element), we expand the velocity and stress in terms of Legendre cardinal functions:

$$v_y = \sum_{i=0}^m h_i(x) (v_y)_i = [h] [v_y] \quad (3)$$

$$\tau_{xy} = \sum_{i=0}^m h_i(x) (\tau_{xy})_i = [h] [\tau_{xy}]$$

The Legendre cardinal functions are written as below:

$$h_i(x) = \frac{(x^2 - 1)L'_m(x)}{m(m+1)L_m(x_i)(x - x_i)} \quad (4)$$

$L_m$  are  $m$ -th order Legendre Polynomials.

where :  $M$  is mass matrix and  $C$  is advection matrix.

$$M = J W; \quad J = \frac{dx}{d\xi}; \quad C = WD \quad (10.a)$$

competition points, the method only requires communication between adjacent elements. No global mesh conversion is required and the process is local. A discontinuous Galerkin method has been developed for the solution of the two-dimensional wave equation. The DG method can be regarded as a cross between finite difference and finite element methods and it has many advantages over the properties of both.

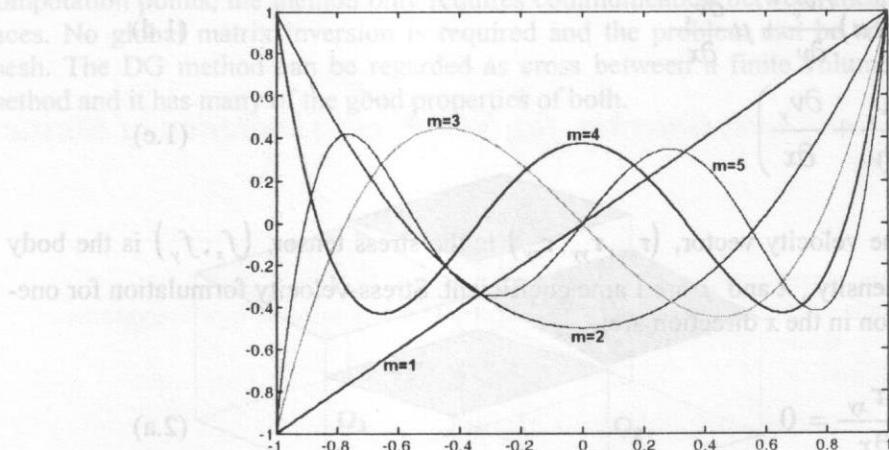


Figure 2. Legendre Polynomials

Figure 1. Mesh and Solution of Discontinuous Galerkin Method

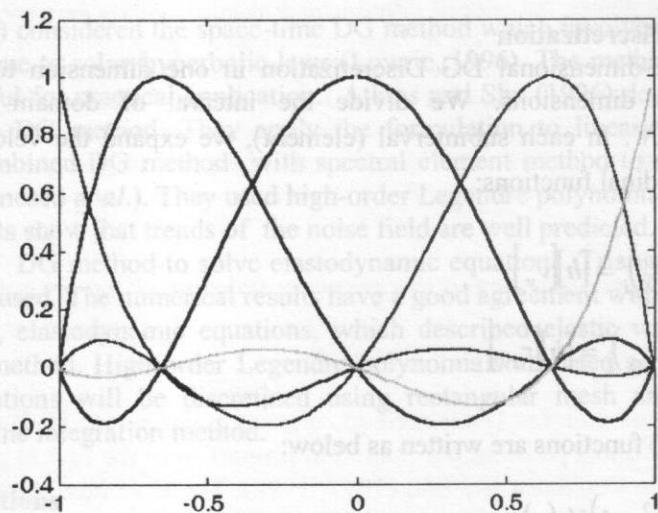


Figure 3. Fourth Order Legendre Cardinal Function

As example, we sample equation (2.a) according Galerkin's procedure using  $[h]$  as testing function:

$$\int_{x_j}^{x_{j+1}} [h]^T [h] \frac{\partial [v_y]}{\partial t} dx - \frac{1}{\rho} \int_{x_j}^{x_{j+1}} [h]^T \frac{\partial [h]}{\partial x} [\tau_{xy}] dx = 0 \quad (5)$$

Integrate by parts the term with spatial derivative:

$$\int_{x_j}^{x_{j+1}} [h]^T [h] \frac{\partial [v_y]}{\partial t} dx - \frac{1}{\rho} \left( - \int_{x_j}^{x_{j+1}} \frac{\partial [h]^T}{\partial x} [h] [\tau_{xy}] dx + [h]^T [h] [\tau_{xy}] \Big|_{x_j}^{x_{j+1}} \right) = 0 \quad (6)$$

We make a modification to approximate  $[\tau_{xy}]_{x_j}^{x_{j+1}}$  by  $[\hat{\tau}_{xy}]_{x_j}^{x_{j+1}}$ , integrate by parts the second term:

$$\int_{x_j}^{x_{j+1}} [h]^T [h] \frac{\partial [v_y]}{\partial t} dx - \frac{1}{\rho} \left( \int_{x_j}^{x_{j+1}} \frac{\partial [h]^T}{\partial x} [h] [\tau_{xy}] dx + [h]^T [h] [\hat{\tau}_{xy} - \tau_{xy}^-] \Big|_{x_j}^{x_{j+1}} \right) = 0 \quad (7)$$

The numerical flux  $(\hat{\tau}_{xy})$  at the interface between elements is approximated by using average flux:

$$\hat{\tau}_{xy} = \frac{1}{2} (\tau_{xy}^+ + \tau_{xy}^-) \quad (8)$$

The (+) notation implies the limit of  $\tau_{xy}$  from outside of the element and the (-) notation implies the limit of  $\tau_{xy}$  from inside of the element.

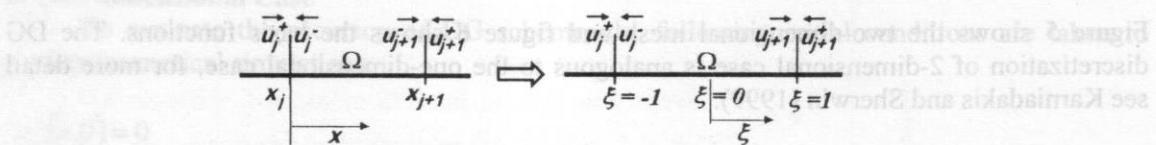


Figure 4. Flux and Local Coordinate

The global coordinate ( $x$ ) are mapped onto local coordinate ( $\xi$ ), each element  $\Omega = [x_j, x_{j+1}]$  is then mapped onto master element  $\Omega = [-1, 1]$ . All integral are evaluated numerically using Gauss Lobatto Legendre (GLL) quadrature. GLL quadrature has accuracy of order  $(2m-2)$ .

The final expressions for the DG discretizations of equation (2.a) is:

$$M \frac{\partial [v_y]}{\partial t} - \frac{1}{\rho} C [\tau_{xy}] - \frac{1}{\rho} \left( [h]^T [h] [\hat{\tau}_{xy} - \tau_{xy}^-] \Big|_{x_j}^{x_{j+1}} \right) = 0 \quad (9)$$

where : M is mass matrix and C is advection matrix.

$$M = JW; J = \frac{dx}{d\xi}; C = WD \quad (10.a)$$

$$\mathbf{D} = \frac{\partial h_j(\xi_i)}{\partial \xi} \begin{cases} \frac{L_m(\xi_i)}{L_m(\xi_j)(\xi_i - \xi_j)} & \text{if } i \neq j \\ 0 & \text{if } i = j, i \neq 0, m \\ \frac{-m(m+1)}{4} & \text{if } i = j = 0 \\ \frac{m(m+1)}{4} & \text{if } i = j = m \end{cases} \quad (10.b)$$

$$\mathbf{W} = \begin{bmatrix} \omega_1 & 0 & - & 0 \\ 0 & \omega_2 & - & 0 \\ - & - & - & - \\ 0 & 0 & - & \omega_m \end{bmatrix} \quad (10.c)$$

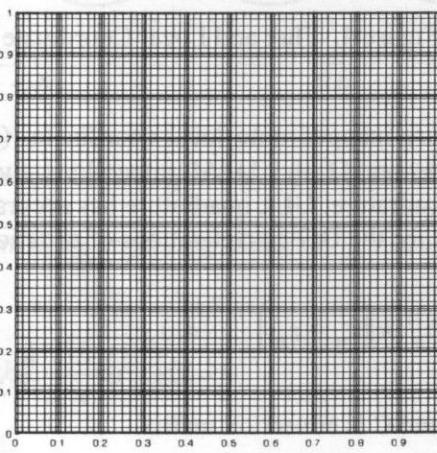
$J$  is the Jacobian  $\mathbf{W}$  is the weight matrix and  $\omega_j$  are the weights and  $\mathbf{D}$  is the differential matrix.

### b. Two Dimensional Discretization

The domain is divided into non-overlapping rectangular elements within which a high order polynomial expansion is used. We mapped the global coordinates  $(x, y)$  onto local coordinates  $(\xi, \eta)$ . The basis (trial or test function) is constructed by taking a product of the one-dimensional basis which can be thought of as one-dimensional tensors. We take expansion of  $v_y$  as example:

$$v_y = \sum_{k=1}^m \sum_{l=1}^m v_{ykl} h_k(\xi) h_l(\eta) \quad (11)$$

Figure 5 shows the two-dimensional mesh and figure 6 shows the basis functions. The DG discretization of 2-dimensional case is analogous to the one-dimensional case, for more detail see Karniadakis and Sherwin (1999).



(8.01)

Figure 5. Two Dimensional Mesh

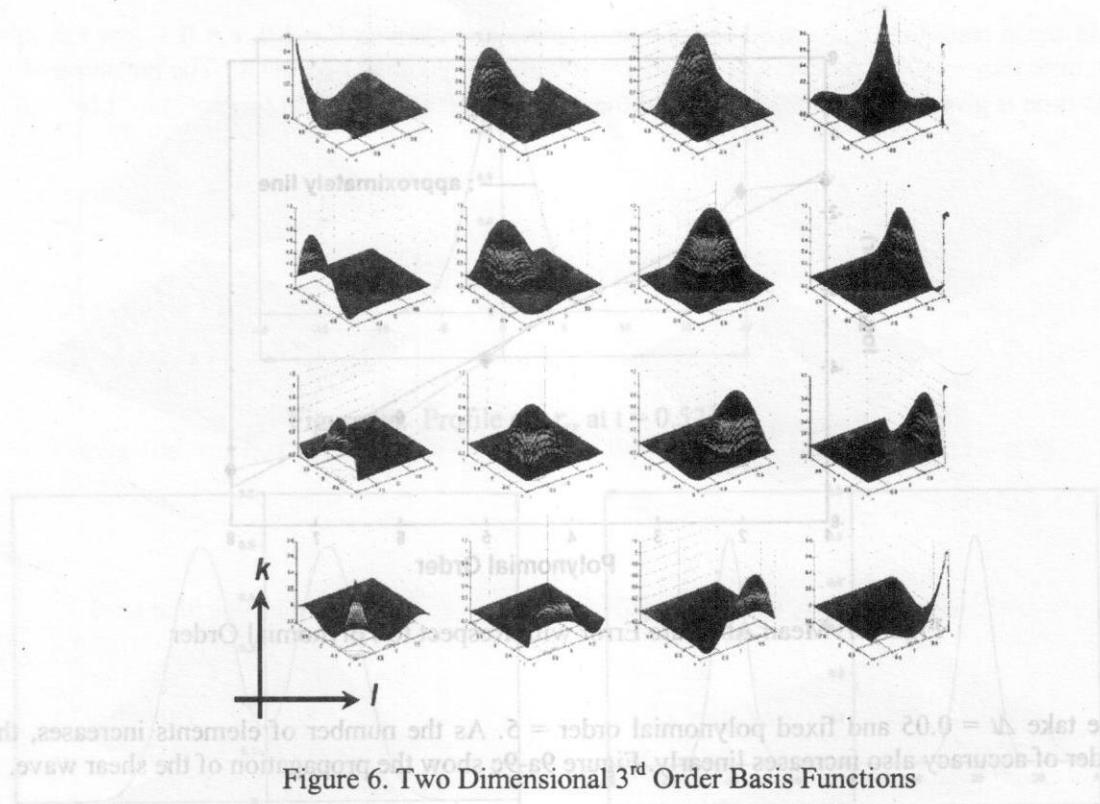


Figure 6. Two Dimensional 3<sup>rd</sup> Order Basis Functions

#### 4. Numerical Results and Discussion

##### a. One dimensional Case

To evaluate the accuracy of DG scheme, the following initial conditions are taken to perform numerical simulations:

$$\begin{aligned} v_y(x, 0) &= 0 \\ \tau_{xy}\left(x, \frac{\Delta t}{2}\right) &= \exp\left(-\ln 2 x^2 / 9\right) ; -100 \leq x \leq 100 \end{aligned} \quad (12)$$

The results are compared to known exact solution, the exact solution in this case is:

$$\tau_{xy}(x, t) = \begin{cases} \exp\left(-\ln 2((x-t)^2)/9\right) + \\ \exp\left(-\ln 2((x+t)^2)/9\right) \end{cases} / 2 \quad (13)$$

We take  $\Delta t = 0.05$  and fixed number of elements = 50. Figure 5 shows the mean absolute error (MAE) versus polynomial order. The MAE is plotted on logarithmic scale and we can deduce exponential (spectral) convergence from approximately straight line on the plot.

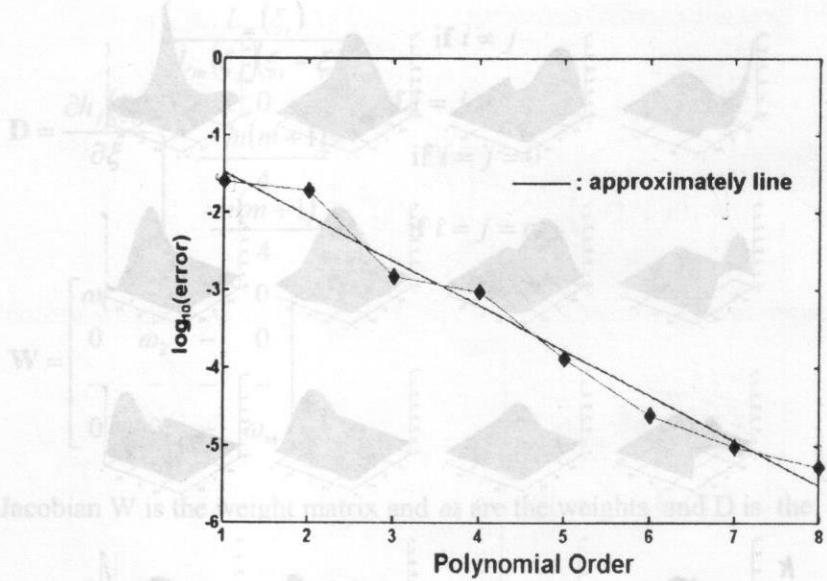


Figure 7. Mean Absolute Error with Respect to Polynomial Order

b. Two Dimensional Mesh: The domain  $\Omega = (0, \pi) \times (0, \pi)$  is discretized into  $n \times n$  quadrilateral elements. We map the element indices onto local indices. The number of nodes per element is 4. The number of nodes per boundary node is 2. The number of boundary nodes is 4. The number of boundary edges is 4. The number of interior nodes is  $(n-1)^2$ . The number of interior edges is  $4(n-1)$ . The number of interior quadrilaterals is  $(n-1)^2$ .

We take  $\Delta t = 0.05$  and fixed polynomial order = 5. As the number of elements increases, order of accuracy also increases linearly. Figure 9a-9c show the propagation of the shear wave as example.

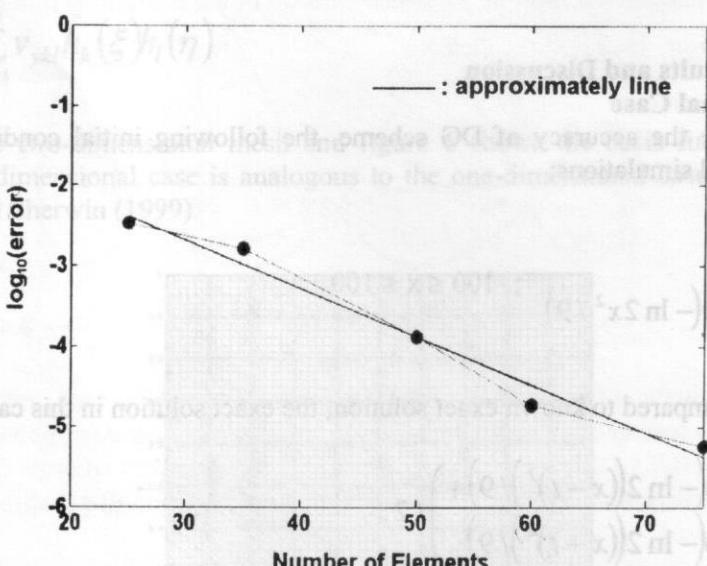


Figure 8. Mean Absolute Error with Respect to Number of Elements

c. Two Dimensional Mesh: The domain  $\Omega = (0, \pi) \times (0, \pi)$  is discretized into  $n \times n$  quadrilateral elements. We map the element indices onto local indices. The number of nodes per element is 4. The number of nodes per boundary node is 2. The number of boundary nodes is 4. The number of boundary edges is 4. The number of interior nodes is  $(n-1)^2$ . The number of interior edges is  $4(n-1)$ . The number of interior quadrilaterals is  $(n-1)^2$ .

Figure 5. Two Dimensional Mesh

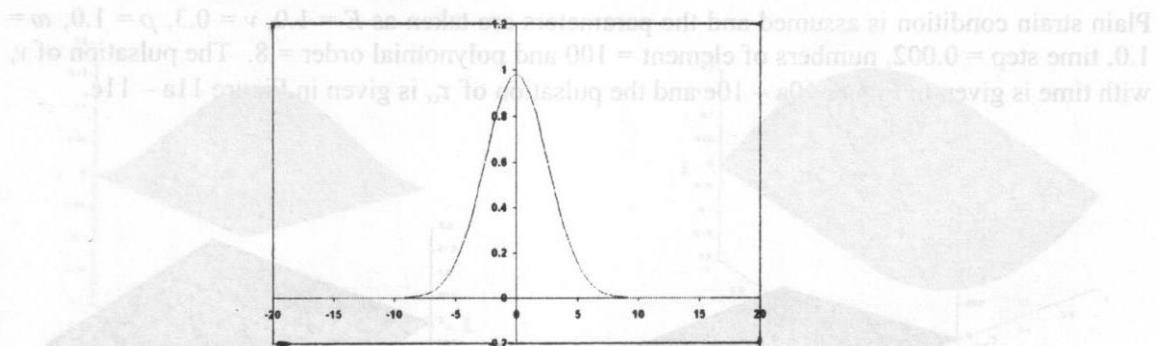


Figure 9a. Profile of  $\tau_{xy}$  at  $t = 0.525$

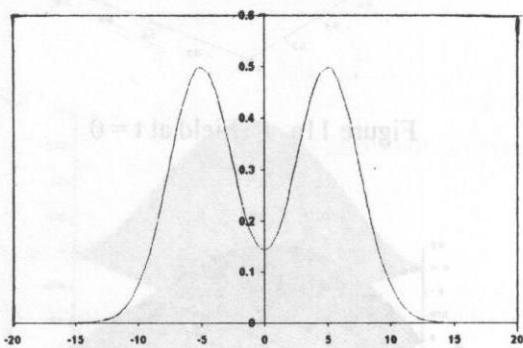


Figure 9b. Profile of  $\tau_x$  at  $t = 5.525$

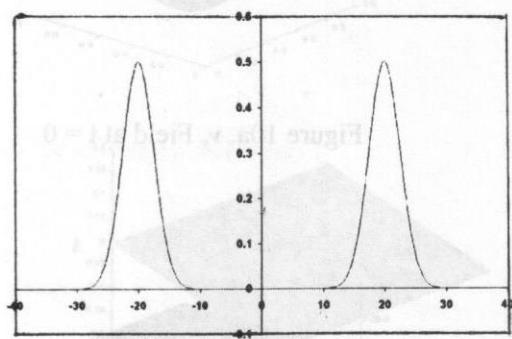


Figure 9c. Profile of  $\tau_x$  at  $t = 20.025$

### b. Two Dimensional Case

We consider a problem defined on the unit square  $[0,1] \times [0,1]$  with  $(v_x, v_y) = 0$  on the boundary. We choose the body force as (Li, 1996):

$$\begin{aligned} f_x &= (\lambda + \mu)(1 - 2x)(1 - 2y)\sin\omega t \\ f_y &= (\rho\omega^2 xy(1-x)(1-y) - 2\mu y(1-y) \\ &\quad - 2(\lambda + 2\mu)x(1-x))\sin\omega t \end{aligned} \quad (14)$$

The exact solutions are:

$$\begin{aligned} v_x &= 0 \\ v_y &= -xy(1-x)(1-y)\cos\omega t \\ \tau_{xx} &= -\lambda x(1-x)(1-2y)\sin\omega t \\ \tau_{yy} &= -(\lambda + 2\mu)x(1-x)(1-2y)\sin\omega t \\ \tau_{xy} &= -\mu y(1-y)(1-2x)\sin\omega t \end{aligned} \quad (15)$$

Plain strain condition is assumed and the parameters are taken as  $E = 1.0$ ,  $\nu = 0.3$ ,  $\rho = 1.0$ ,  $\omega = 1.0$ , time step = 0.002, numbers of element = 100 and polynomial order = 8. The pulsation of with time is given in Figure 10a – 10e and the pulsation of  $\tau_{xx}$  is given in Figure 11a – 11e.

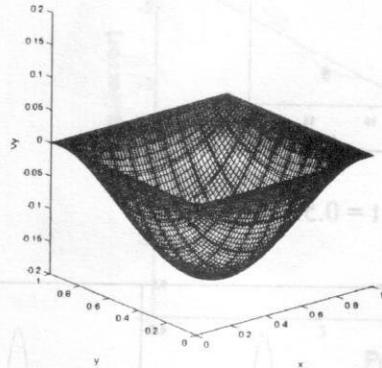


Figure 10a.  $v_y$  Field at  $t = 0$

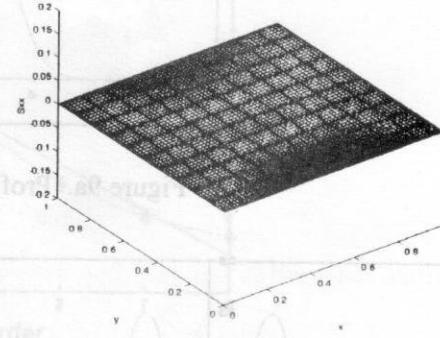


Figure 11a.  $\tau_{xx}$  Field at  $t = 0$

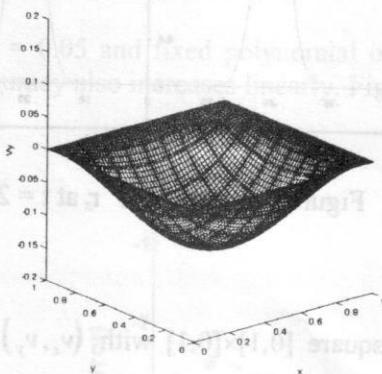


Figure 10b.  $v_y$  Field at  $t = 0.25$

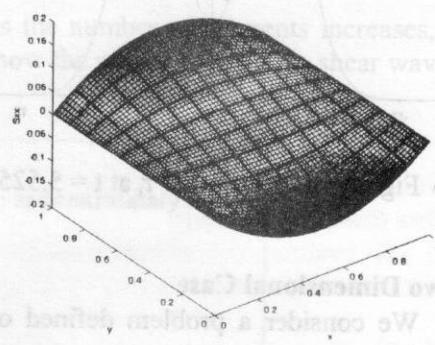


Figure 11b.  $\tau_{xx}$  Field at  $t = 0.25$

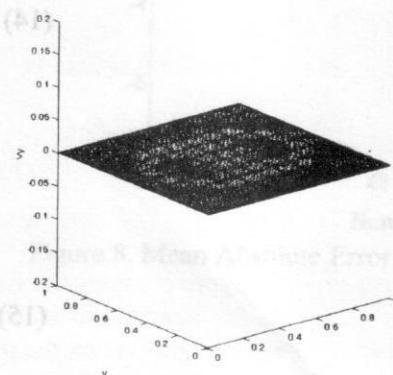


Figure 10c.  $v_y$  Field at  $t = 0.5$

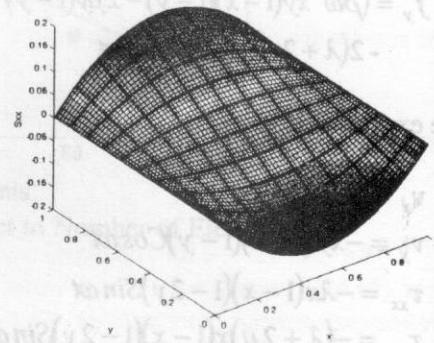


Figure 11c.  $\tau_{xx}$  Field at  $t = 0.5$

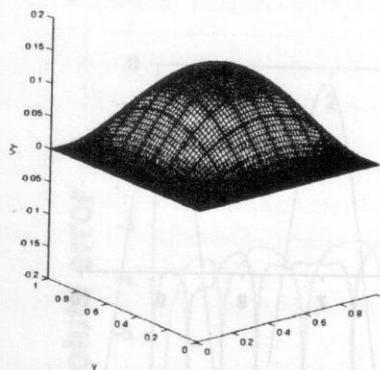


Figure 10d.  $v_y$  Field at  $t = 0.75$

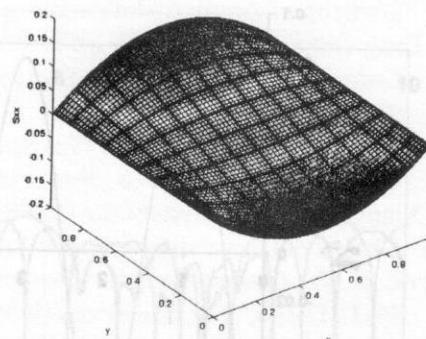


Figure 11d.  $\tau_{xx}$  Field at  $t = 0.75$

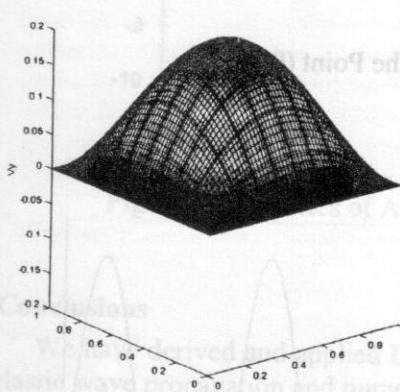


Figure 10e.  $v_y$  Field at  $t = 1.0$

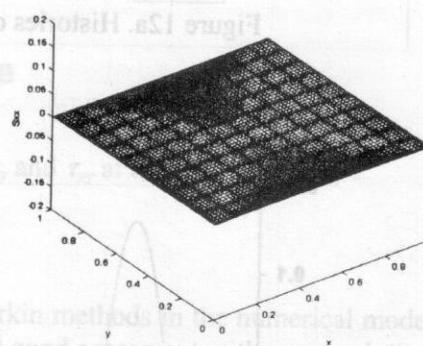


Figure 11e.  $\tau_{xx}$  Field at  $t = 1.0$

Comparisons with the exact solutions are shown in figure 12a and 12b for  $v_y$  and  $\tau_{yy}$  profile, very good agreements are found. Figure 13 shows that trends of DG errors almost constant in time, the growth of error (dispersive & dissipation error) in DG method can be reduced by using high-order basis.

- Atkins, Harold L. and Shu, Chi Wang, 1995, *Quadrature-Free Implementation of Discontinuous Galerkin for Hyperbolic Equations*, NASA AIAA paper 95-1683.
- Atkins, Harold L., 1997, *Continued Development of The Discontinuous Galerkin Method for Computational Aerodynamics*, ICASE Report 97-32.
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Plain strain condition is assumed and the parameters are taken as  $E = 1.0$ ,  $\nu = 0.3$ ,  $\rho = 1.0$ ,  $\omega = 1.0$ , time step = 0.001, numbers of element = 100 and polynomial order = 8. The pulsation of  $v_y$  with time is given in Figure 12a,  $\tau_{yy}$  = 1.0 and the pulsation of  $v_x$  is given in Figure 11a – 11c.

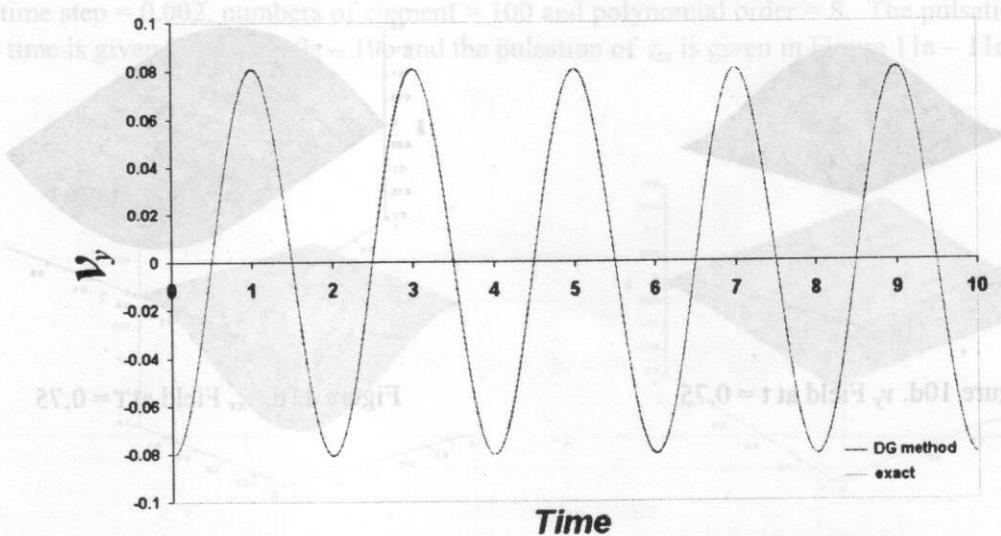


Figure 12a. Histories of  $v_y$  Responses at the Point (0.4,0.4)

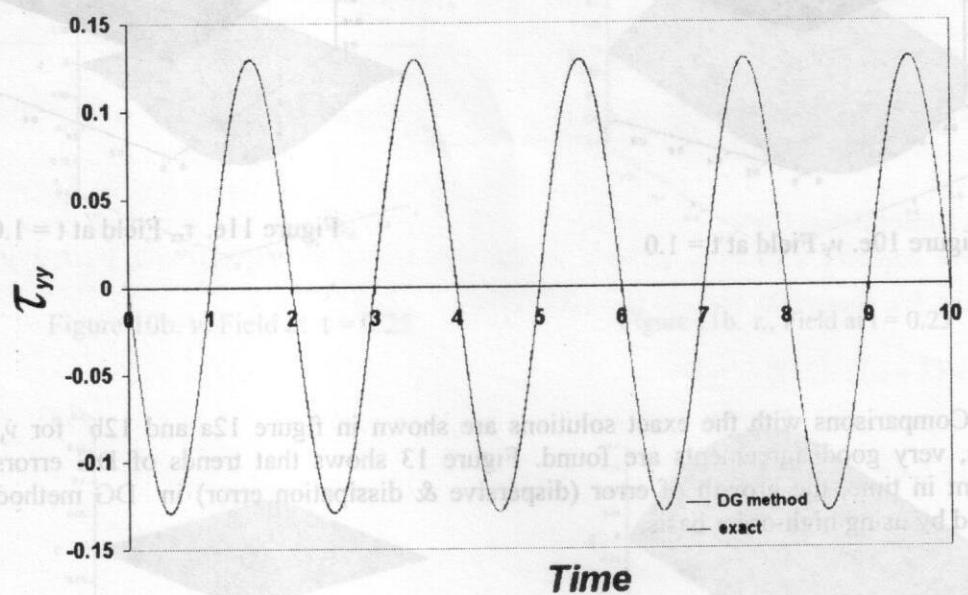


Figure 12b. Histories of  $\tau_{yy}$  Responses at the Point (0.4,0.4)

Figure 10c.  $v_y$  Field at  $t = t_0$

Figure 11c.  $\tau_{yy}$  Field at  $t = 0.5$

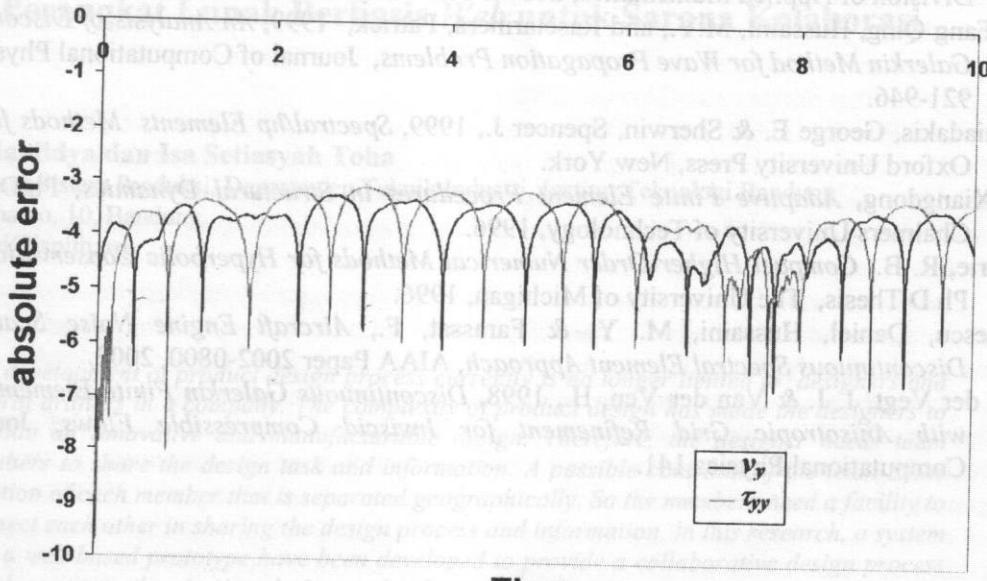


Figure 13. Histories of Absolute Errors of  $v_y$  and  $\tau_{yy}$  at the Point (0.4,0.4)

## 5. Conclusions

We have derived and applied Discontinuous Galerkin methods in the numerical modeling of elastic wave propagation and numerical results have a good agreement with exact solution. In simple domain, the exponential convergence can be achieved by either increasing the number of elements, called *h-refinement*, or by increasing the polynomial order of a fixed number of elements, called *p-refinement*. For future research, we plan to extend the DG method for solving problems with irregular domain and apply *hp adaptive* technique to increase the accuracy and to reduce computational costs.

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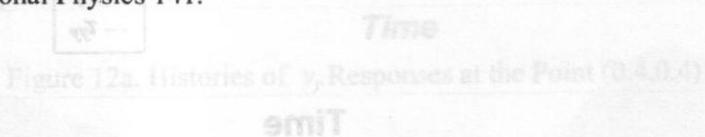


Figure 12a. Histories of  $v_y$  Responses at the Point  $(0.4, 0, 0)$

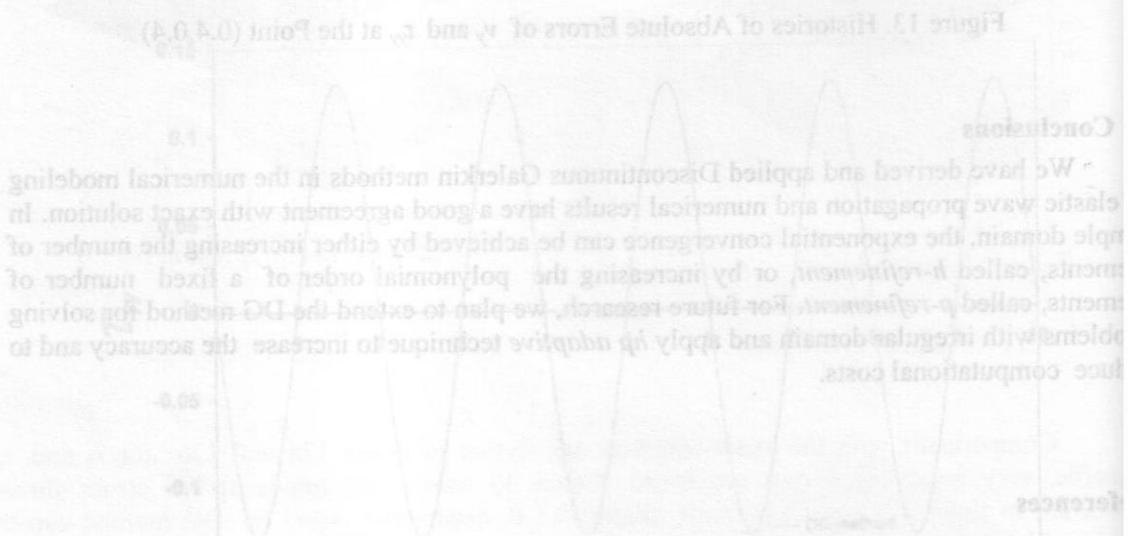


Figure 13. Histories of Aperture Plots of  $v_y$  and  $v_z$  at the Point  $(0.4, 0, 0)$

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