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Numerical Solution of Steady State Free Convection using Unstructured Discontinuous Galerkin Method

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Abstract-The development of discontinuous Galerkin method in order to solve the steady state of natural convection problem is presented. In the present method, the collocated nodes were used, whereas the velocity, pressure and temperature fields were located on the same nodes. The pseudo-unsteady and explicit low storage fourth orders Runge-Kutta method scheme were used for the time integration. Next, the discontinuous Galerkin finite element and unstructured mesh were employed to calculate the spatial discretization. The difficulty relates to the pressure can be overcome by using an artificial compressibility method. Solutions to the 2-D natural convective flow in concentric annulus have been obtained and compared with available results. The comparisons showed a good agreement.

Key words: Natural convection, discontinuous Galerkin, artificial compressibility

1. INTRODUCTION

The free convective flow, especially in a concentric annulus, can be found in many industrial applications. Such problem commonly occurs in the electrical, nuclear energy fields, as well as in solar energy and thermal storage system. Due to the flow behavior in this problem are very complicated; therefore, it is necessary to clarify the flow structure and the mechanism in details to establish an improved design technique. The numerical method has a powerful ability to investigate the flow behavior and to clarify the predominant factors that influence the flows.

Extensive works in this field have been performed in the past. Takata et al. (1984) used finite difference method to solve 3-D natural convection in an inclined cylindrical annulus. Shu et al. (1999,2000) used Generalized Differential Quadrature (GDQ) to simulate the natural convection in concentric and arbitrarily eccentric annulus. Recently, Shi et al. (2006) proposed a finite difference-based Lattice Boltzmann for solving natural convection in concentric annulus.

The above proposed methods have the difficulties in solving problems with complicated geometries. Ramaswamy (1988) employed finite element method to solve natural convection problems. Finite element method can handle complicated domains easily. Manzari (1998) used standard galerkin finite element to simulate forced and natural convection heat transfer. Explicit Runge Kutta scheme was used in the calculation of time domain. The procedure is stabilized using an artificial dissipation technique. The continuity equation is modified by employing an artificial compressibility concept in order to couple the pressure and velocity fields of the fluid. Meanwhile a general solution to solve the relating problem is still unclear.

The present study, we concern with the numerical prediction of the natural convection in concentric annulus. The general method is pseudo-unsteady and uses a low storage fourth order Runge Kutta for the time integration as reported by Kennedy and carpenter (1994) . The spatial discretization uses a high order nodal discontinuous Galerkin and unstructured mesh. The difficulty relating to the pressure can be overcome by using an artificial compressibility method.

2. PROBLEM DESCRIPTION

In this problem, we considered a Newtonian fluid of kinematic viscosityν, thermal diffusivity α and thermal expansion coefficient β enclosed in a concentric annulus with inner radius $r_i = 0.625$ and outer radius $r_i = 1.625$. The vertical (opposite to the gravity vector) and horizontal axis in a coordinate system is defined respectively as y-axis and x-axis. Constant uniform temperatures T_h and T_c $(T_h > T_c)$ are imposed at the inner and outer walls, respectively.

An appropriate scaling for the flow regime is characterized by the following reference quantities (Le Quere, 1991): $L = r_o - r_i$ for length; $V = (\alpha/L)Ra^{0.5}$ for velocity, where Ra is the Rayleigh number $(g\beta\Delta TL^3/\nu\alpha)$; $t = (L^2/\alpha)Ra^{-0.5}$ for time and the scaled temperature θ is defined as: $(T - T_r)/(T - Tc)$.

By using the above reference quantities, the governing equations is defined as follow:

$$\frac{\partial \mathbf{q}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{S} \qquad (1)$$

$$
\mathrm{q}=\begin{bmatrix} p \\ u \\ v \\ \theta \end{bmatrix} \qquad \mathbf{F}=\begin{bmatrix} \varepsilon^2 u \\ uu+p-\dfrac{\Pr}{\mathrm{Ra}^{0.5}}\dfrac{\partial u}{\partial x} \\ uv-\dfrac{\Pr}{\mathrm{Ra}^{0.5}}\dfrac{\partial v}{\partial y} \\ u\theta-\dfrac{1}{\mathrm{Ra}^{0.5}}\dfrac{\partial \theta}{\partial x} \end{bmatrix}
$$

$$
\mathbf{G}=\begin{bmatrix} \varepsilon^2 v \\ uu+p-\dfrac{\Pr}{\mathrm{Ra}^{0.5}}\dfrac{\partial u}{\partial x} \\ uv-\dfrac{\Pr}{\mathrm{Ra}^{0.5}}\dfrac{\partial v}{\partial y} \\ v\theta-\dfrac{1}{\mathrm{Ra}^{0.5}}\dfrac{\partial \theta}{\partial y} \end{bmatrix} \qquad \mathbf{S}=\begin{bmatrix} 0 \\ 0 \\ \Pr\theta \\ 0 \end{bmatrix}
$$

where u and v are velocity components, p is the deviation from hydrostatic pressure divided by the mean density multiplied by V^2, and ε is artificial compressibility coefficient. Pr is the Prandtl number (ν/α).

The boundary conditions are as follows

- u, v and w = 0 at the walls
- θ = 0.5 on $r=r_i$ and θ = -0.5 on $r=r_o$

Here, pressure and velocity fields are set to zero as initial conditions.

3. DISCRETIZATION

The spatial derivatives are discretized by using a discontinuous galerkin method. The simplified of Eq.(1) according to Galerkin's procedure using the same basis function ϕ within each element is defined below:

$$
\left(\phi,\frac{\partial \mathrm{q}}{\partial t}+\mathbf{A}\frac{\partial \mathrm{q}}{\partial x}+\mathbf{B}\frac{\partial \mathrm{q}}{\partial y}\right)=0
$$
$$
\Leftrightarrow\left(\phi,\frac{\partial \mathrm{q}}{\partial t}\right)_{\Omega}+\left(\phi,\mathbf{A}n_x\,\mathrm{q}+Bn_y\,\mathrm{q}\right)_{\partial\Omega}-\left(\frac{\partial}{\partial x}(\mathbf{A}\phi),\mathrm{q}\right)_{\partial\Omega}
$$
$$
-\left(\frac{\partial}{\partial y}(\mathbf{B}\phi),\mathrm{q}\right)_{\Omega}=0 \tag{2}
$$

Here $(.,.)$ represents the normal $2\,L$ inner product and the second term is flux vector. The mathematical manipulation of the flux vector is as below:

$$
\left(\phi,\frac{\partial \mathrm{q}}{\partial t}+\mathbf{A}\frac{\partial \mathrm{q}}{\partial x}+\mathbf{B}\frac{\partial \mathrm{q}}{\partial y}\right)_{\Omega}+
$$
$$
\left(\phi,\mathbf{A}n_x+\mathbf{B}n_y\right)\left(\hat{\mathrm{q}}-\mathrm{q}^-\right)_{\partial\Omega}=0 \tag{3}
$$

where,

$$
\mathbf{q}^-\big|_{\partial\Omega}=\hat{\mathbf{q}}^-\left(\mathbf{q}^-,\mathbf{q}^+\right)\big|_{\partial\Omega}
$$

In this problem, the numerical flux vector is calculated by using the Lax-Friedrich flux.

$$
\left(\phi,\frac{\partial \mathrm{q}}{\partial t}+\mathbf{A}\frac{\partial \mathrm{q}}{\partial x}+\mathbf{B}\frac{\partial \mathrm{q}}{\partial y}\right)_{\Omega}+
$$
$$
\left(\phi,\frac{(\mathbf{A}n_x+\mathbf{B}n_y)}{2}[\mathrm{q}]\right)_{\partial\Omega}-\left(\phi,\frac{\lambda}{2}[\mathrm{q}]\right)_{\partial\Omega}=0 \tag{4}
$$

where $[\mathrm{q}]=\mathrm{q}^+-\mathrm{q}^-$ and $\lambda=\max\left(\sqrt{(u^2+v^2)+\varepsilon^2}\right)$ is the largest wave speed. Here, we took the Kornwinder Dubiner function on straight sided triangle as the basis written in equation 5 (see Figs. 1 and 2):

$$
\phi_{ij}(r,s)=
$$
$$
\sqrt{\frac{2i+1}{2}}\sqrt{\frac{2i+2j+2}{2}}P_i^{0,0}\left(\frac{2(1+r)}{(1-s)}-1\right)P_j^{2i+1,0}(s) \tag{5}
$$

where, $P^{\alpha,\beta}$ is orthogonal Jacobi polynomial

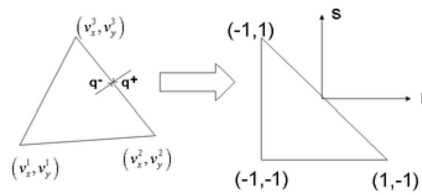

Figure 1.
Coordinate Transformation

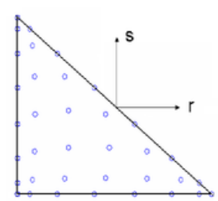

Figure 2.
Seventh order Gauss Lobatto Quadrature Nodes

The vector $\mathbf{q}=(p\ \ u\ \ v\ \ \theta)^T$ is expanded using equation (5) as follows:

$$
p(r,s)=\sum_{i=0}^{N}\sum_{j=0}^{N-i}\phi_{ij}(r,s)\hat{p}_{ij} \tag{6}
$$

$$
p(r_n,s_n)=\sum_{m=1}^{m=M}\mathbf{V}_{nm}\hat{p}_m
$$
$$
\hat{p}_m=\sum_{m=1}^{m=M}\left(\mathbf{V}^{-1}\right)_{mj}p(r_j,s_j) \tag{7}
$$

$$\frac{\partial p}{\partial r}(r,s) = \sum_{i=0}^{N}\sum_{j=0}^{N-i}\frac{\partial \phi_{ij}}{\partial r}(r,s)\hat{p}_{ij} = \hat{\mathrm{D}}^{r}\mathbf{V}^{-1}p(r,s) \tag{8}$$

$$\frac{\partial p}{\partial s}(r,s) = \sum_{i=0}^{N}\sum_{j=0}^{N-i}\frac{\partial \phi_{ij}}{\partial s}(r,s)\hat{p}_{ij} = \hat{\mathrm{D}}^{s}\mathbf{V}^{-1}p(r,s)$$

$$\hat{\mathbf{D}}^{r} = \frac{\partial \phi}{\partial r}$$

$$\hat{\mathbf{D}}^{s} = \frac{\partial \phi}{\partial s}$$

where $\mathbf{V}_{ij}$ and N are Vandermonde matrix dan the order of Jacobi polynomial respectively.

The second order terms in equation (1) are solved using local discontinuous galerkin (LDG) method with central fluxes as suggested by Warburton (2003). The semi discrete Eq. (8) is integrated in time marching by using five stage of fourth order 2N-storage Runge-Kutta scheme as developed by Carpenter & Kennedy (1994). The final equations are found as written in Eqs. (9) and (10) for the heat rate and the 5-stage of 2N-storage Runge-Kutta algorithm respectively.

$$\frac{d\mathbf{q}}{dt} = L[t,\mathbf{q}(t)] \tag{9}$$

$$\begin{aligned} d\mathbf{q}_j &= A_j d\mathbf{q}_{j-1} + hL(\mathbf{q}_j) \\ \mathbf{q}_j &= \mathbf{q}_{j-1} + B_j + d\mathbf{q}_j \end{aligned} \tag{10}$$

where h is the time step. The vectors A and B are the coefficients that will be used to determine the properties of the scheme.

The algorithm is implemented as follows.

1. Define the initial and boundary conditions for the pressure, velocities and temperature (p^n, u^n, v^n, w^n, θ^n).
2. Solve the equation (1) by Runge Kutta method to obtain u^{n+1}, v^{n+1} w^{n+1} and θ^{n+1}.
3. Calculate $error = \frac{\sum|\theta^{n+1} - \theta^{n}|}{\sum nodes}$, if error less than tolerance or if iterations is equal to the limit, the calculation will be finished. In addition, the results have no a physical meaning, if the steady state can not be achieved.

4. RESULTS

The implementation of numerical calculation was done by using the Matlab on 1600 MHz Centrino Duo personal computer. The Rayleigh (Ra) numbers were varied from $2.38\text{x}10^3$ to $1.02\text{x}10^5$. We used the single mesh for all the ranges of Ra numbers. The mesh consisted of 478 elements. For all the calculation, we took a fixed order of polynomial N=4, Prandtl number is 0.717 and fixed artificial compressibility coefficients is equal to unity.

The calculation result of the isotherms at steady states for different Ra and Pr numbers are shown in Fig. 4.

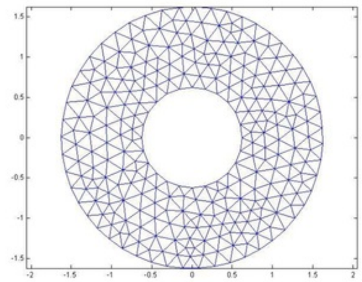

Figure 3.
Mesh

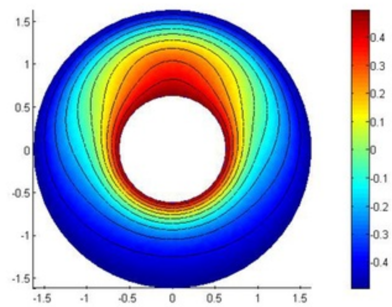

Fig. 4a. Isotherm of Ra=$2.38\text{x}10^3$

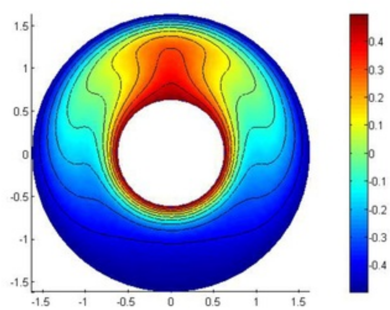

Fig. 4b. Isotherm for Ra=$9.50\text{x}10^3$

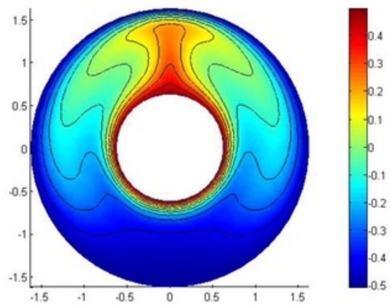

Fig. 4c. Isotherm of Ra=3.20x10^4

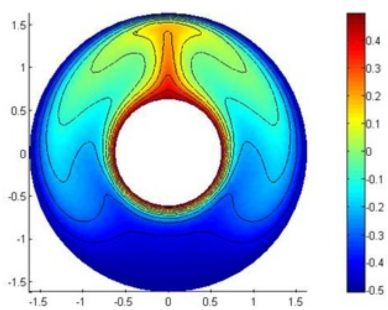

Fig.4d. Isotherm of Ra=6.19x10^4

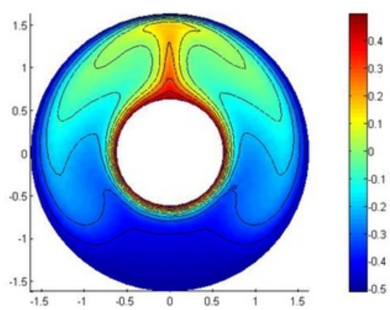

Fig. 4e. Isotherm of Ra=1.02x10^5

From fig.(4) it is shown clearly that at low Ra number, the fluid motion driven by the buoyancy force is slow, leading to the strong diffusion. Consequently, the corresponding isothermals exhibit rather slight difference as compared to those of pure heat conduction between the annulus. On the other hand, when Ra number is increased, the buoyancy force accelerates the circulation of fluid flow and natural convection is significantly enhanced. The above-discussed behaviors are also found in the previous numerical study as reported by Shi et al. (2005).

Regarding the effect of an increasing of the Rayleigh number on the mean value of Nussel number is shown in Table 1. The results indicated that a good agreement among the present numerical results, the previous proposed experimental data, and the previous numerical resulted by the finite difference Lattice Boltzmann (FDLB) method (see Shi et al., 2006). Here, the mean value of Nu was computed by the bellow equations:

$$Nu_{mean} = \frac{1}{2}\left(Nu_{inner} + Nu_{outer}\right) \quad (11)$$

$$Nu_{inner} = -\frac{1}{2\pi}\oint_{S_{inner}}\left(\frac{\partial\theta}{\partial x}n_x + \frac{\partial\theta}{\partial y}n_y\right)dS \quad (12)$$

$$Nu_{outer} = -\frac{1}{2\pi}\oint_{S_{outer}}\left(\frac{\partial\theta}{\partial x}n_x + \frac{\partial\theta}{\partial y}n_y\right)dS \quad (13)$$

Table 1
The mean value of Nusselt number

Ra	**Present work**	**Experimental* (Kuhn & Goldstein)**	**FDLB***
3.28x10^3	1.363	1.38	1.320
9.50x10^3	2.055	2.01	1.999
3.20x10^4	2.861	2.89	2.911
6.19x10^4	3.318	3.32	3.361
1.02x10^5	3.741	3.66	3.531

*** adopted from: Shi et al. (2006)**

6. CONCLUSSION

In this paper, we have presented a discontinuous galerkin method for solving natural convection in artificial compressibility formulation. The method is fully explicit and exhibits good numerical stability. The numerical results have a good agreement with the proposed experimental and numerical results reported in the previous studies.

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