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The 11th International Conference on QiR (Quality in Research)

3 - 6 August 2009 at the Faculty of Engineering, University of Indonesia

as

Presenter

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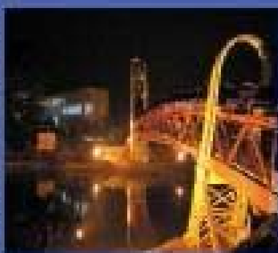

Faculty of Engineering
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Welcome from the Rector of University of Indonesia

I am honoured to have the opportunity to officially welcome you to the 11th International Conference on QiR (Quality in Research) 2009. The conference provides an excellent forum for engineering professionals, business executives, industry practitioners, and academicians to exchange ideas and to share their experience, knowledge and expertise with each other. I believe the participants will also learn about the latest trends in the development of new tools, knowledge and skills in various engineering design and technology.

As we agree that engineering products or projects bring together resources, skills, technology and ideas to achieve business objectives and deliver business benefits and it comes in all shapes and sizes from the simple and straight-forward to the large and unmentionably complex, thus we need an application of knowledge, skills, tools, and techniques necessary to develop and successfully execute the products or projects plan so it will meet or exceed our customer and stakeholder needs and expectations.

The ultimate concern of engineering product or project is three-fold: the product/project meeting its targets and purposes, the product/project on schedule, and the product/project cost within budget. As the Indonesian economy is growing, local enterprises are obliged to upgrade their skills in innovation and product design. There are also increasingly aware of the importance of professionalism in various engineering areas for industries needs and many professionals are keen to upgrade their capability.

Having said that, I hope this conference can be a kick-off for strengthened our action and partnerships on creating a platform for us; national and international thinkers, academics, government officials, business executives and practitioners, to present and discuss the pivotal role of engineers in the achievement of excellence organizations.

I am sure you will find the 11th International Conference on QiR (Quality in Research) 2009 both informative and stimulating. I would like to thank the Faculty of Engineering, University of Indonesia for organizing this meaningful and timely event, and the supporting organizations for their participation and contributions. With this, I wish you all a fruitful conference. Thank you.

Prof. Dr. der. Soz. Gumilar Rusliwa Somantri
Rector of University of Indonesia
University of Indonesia

Welcome from Dean of Engineering University of Indonesia

On behalf of the Faculty of Engineering, University of Indonesia, it is my greatest pleasure to extend our warmest welcome to all of you to the 11th International Conference on QiR (Quality in Research) 2009. As we know that this conference is conducted to cover a wide range of engineering design and technology issues. I hope these four days of the conference will be spent in interesting discussion and exchange of ideas. I also hope this conference would be able to provide a state-of-the-art information and knowledge in the challenging world of engineering design and technology. The growing success of our institutions and expertise should urge us to develop our competitive capabilities, especially as we face certain challenges which would be overcome with more hard work and working together hand by hand. We will work together to develop a common path and collaboration opportunities for future action and research on multi disciplinary engineering areas.

I am delighted that you have accepted our invitation to this conference in such large numbers as indicated that we will have many international keynote speakers' lectures and papers from various countries to be presented and discussed during these two days conference. We will explore various engineering techniques and tools in various industries that can be used to build better stakeholder performance and relationships, to enable us to create wealth through innovation, to promote productivity through technology, and to foster our collaboration.

I would like to thank you to our sponsors, supported bodies and various contributors for their generous support of this conference. I would also like to thank our distinguished speakers for agreeing to share their insights with us. To our friends from overseas and other provinces of Indonesia, I would also like to extend a warm welcome to you and wish you an enjoyable stay in Jakarta. Last but not least, I would invite you to join me in thanking the committed staff that made this conference happen and to make it a success.

I wish you a very pleasant stay here in Jakarta and a successful and productive discussion at the conference. Thank you.

Prof. Dr. Bambang Sugiarto
Dean of Engineering University of Indonesia

Welcome from the QiR 2009 Organizing Committee

On behalf of the Organizing Committee, it is my greatest pleasure to extend our warmest welcome to all of you to the 11th International Conference on QiR (Quality in Research) 2009.

I am sure that you will all find this conference stimulating and rewarding. As we are aware of, the impact of globalization has resulted in a very competitive business environment that makes the fulfillment of customer/clients' ever-sophisticated project or product or service needs most challenging. Without any doubt, a good engineering design and technology is powerful in helping our industries to enhance their productivity and competitiveness. Thus, it is our aim and hope that the conference would be able to provide an international forum for exchange knowledge and research expertise as well as to create prospective collaboration and networking on various fields of engineering and architecture.

With its continuous presence in the last 11 years, QiR has become an icon of Faculty of Engineering University of Indonesia in serving the objectives to provide engineering excellence for both national and international needs. The QiR 2009 consists of 2 special issues and 4 symposia covering almost all aspect in engineering, design and architecture. I am delighted to inform you that we have such large numbers of participants today as indicated that we will have 7 keynote speakers' presentation and more than 240 papers from various countries to be presented and discussed during these two days conference. We are fortunate to have a lot of good quality of papers that belongs to:

70 papers on Radio Frequency Identification (RFID) as a Bridge between Computing and Telecommunication
36 papers on Green Infrastructure for Sustainable Development and Tropical Eco-Urbanism.
56 papers on Industrial Engineering Approach for Productivity Improvement
56 papers on Advanced Materials and Processing
30 papers on Energy Conservation through Efficiency in Design and Manufacturing

I would like to thank you to various contributors, speakers and participants for your generous support of this conference. It is my pleasant duty to thank all the members of Organizing Committee and the International Board of Reviewers for their advices and help. We are grateful to all the Sponsors, Supporters and Exhibitors for their spontaneous response and encouragement by way of committing funds and extending help in kind.

I would like to sincerely thank the Dean of Engineering, for fully supporting the Committee and providing all supports to make this conference happen and to make it a success.

I wish you a very pleasant stay here in Jakarta and finally, let me wish all of you a meaningful and fruitful conference. Thank you and hope to see you again in QiR 2011.

Dr. Bondan T. Sofyan
Organizing Chairperson of QiR 2009

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Discontinuous Galerkin Method for Solving Steady and Unsteady Incompressible Navier Stokes Equations

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ABSTRACT

Purpose: *To develop high order discontinuous galerkin method for solving steady and unsteady incompressible flow based on artificial compressibility method.*
Design/methodology/approach: *This paper uses discontinuous galerkin finite element procedure which is based on the artificial compressibility technique in connection with a dual time stepping approach. A second order implicit discretization is applied to achieve the required accuracy in real time while an explicit low storage fourth order Runge Kutta scheme is used to march in the pseudo-time domain. A nodal high order discontinuous galerkin finite element is used for the spatial discretization.*
Findings: *Provides stable and accurate methods for solving incompressible viscous flows compared with previous numerical results and experimental results.*
Research limitations: *It is limited to two-dimensional steady and unsteady laminar viscous flow.*
Practical implications *– A very useful source of information and favorable advice for people is applied to piping system and low speed aerodynamics.*
Originality*: This works presents an extension of the previous work of [1] and [2] to time and space accurate method for solving unsteady incompressible flows.*

Keywords: *steady and unsteady flows, discontinuous galerkin, artificial compressibility.*

1. INTRODUCTION

Numerical Solutions of the incompressible Navier-Stokes equations are of interest in many engineering applications. Problems which can be addressed by incompressible Navier-Stokes equations include internal flows, hydrodynamics flows, low speed aerodynamics and external flows. There is a continuing interest in finding solution methodologies which will produce accurate results in time and space. The problem of coupling changes in the velocity field with changes in pressure field while satisfying the continuity equation is the main difficulty in obtaining solutions to the incompressible Navier-Stokes.

There are two types of method using primitive variables which have been developed to solve the equations. The first type of methods can be classified as pressure-based methods. In these methods, the pressure field is solved by combining the momentum and mass continuity equations for form a pressure or pressure-correction equation. The second type of methods employs the artificial compressibility (AC) formulation. This idea was first introduced by Chorin [3] and extensively used by other researchers since. In this method, a pseudo-temporal pressure terms is added to continuity equation to impose the incompressibility constraint. Several authors have employed this method successfully in computing unsteady problems.

The original version of the AC method is only accurate for steady-state solutions to the incompressible flows [4, 5], however there are some efforts which conducted to solve unsteady flows using dual time stepping AC method. Reference [6] used third order flux difference technique for convective terms and second-order central difference for viscous terms. The semi discrete equations are solved implicitly by using block line-relaxation scheme. Reference [1] used finite element method for spatial discretization. A second-order discretization is employed in real time while an explicit multistage Runge Kutta is used to march in pseudo time domain.

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This work presents an extension of the previous work of [1, 2] to time and space accurate method for solving unsteady incompressible flows. A nodal high order discontinuous galerkin finite element is used for the spatial discretization a second order implicit discretization is applied to achieve the required accuracy in real time while an explicit low storage fourth order Runge Kutta scheme is used to march in the pseudo-time domain. The computed results show accuracy of the code by presenting the steady and unsteady flow past a 2-dimensional circular cylinder.

2. PROBLEM DESCRIPTION

Artificial compressibility method is introduced by adding a time derivative of pressure to the continuity equation. In the steady-state formulation, the equations are marched in a time-like fashion until the divergence of velocity vanishes. The time variable for this process no longer represents physical time. Therefore, in the momentum equations t is replaced with τ , which can be thought of as an artificial time or iteration parameter [7]. As a result, the governing equations can be written in the following form:

$$\frac{\partial \mathbf{U}}{\partial \tau}+\frac{\partial \mathbf{F}^j}{\partial x_j}+\frac{\partial \mathbf{G}^j}{\partial x_j}=0 , \quad j=1,2 \qquad (1)$$

Where

$$\mathbf{U}=\begin{bmatrix} p \\ u_1 \\ u_2 \end{bmatrix} \quad \mathbf{F}^j=\begin{bmatrix} \varepsilon^2 u_j \\ u_1 u_j + p\delta_{1j} \\ u_2 u_j + p\delta_{2j} \end{bmatrix} \qquad (2)$$

$$\mathbf{G}^j=\begin{bmatrix} 0 \\ \tau_{1j} \\ \tau_{2j} \end{bmatrix} \quad \tau_{ij}=\frac{1}{\mathrm{Re}}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$$

In these equations, τ is the artificial time variable, u_i is the velocity in direction x_i, p is the pressure, τ_{ij} is stress tensor, ε is an artificial compressibility parameter, δ_{ij} is Kronecker delta and Re is the Reynolds number.

The extension of artificial compressibility method to unsteady flow is introduced by adding physical time derivative of velocity components to 2 momentum equations in Equations (2) [6, 1]. The obtained equations can be written as:

$$\frac{\partial \mathbf{U}}{\partial \tau}+\mathbf{I}^M\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial \mathbf{F}^j}{\partial x_j}+\frac{\partial \mathbf{G}^j}{\partial x_j}=0 , \quad j=1,2 \qquad (3)$$

Where
$$\mathbf{I}^M=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. DISCRETIZATION

The spatial derivatives are discretized by using a discontinuous galerkin method. The simplified of Eq. (3) according to Galerkin's procedure using the same basis function ϕ within each element is defined below:

$$\begin{aligned} &\left(\phi,\frac{\partial \mathbf{U}}{\partial \tau}+\mathbf{I}^M\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial \mathbf{F}^j}{\partial x_j}+\frac{\partial \mathbf{G}^j}{\partial x_j}\right)=0 \\ &\Leftrightarrow\left(\phi,\frac{\partial \mathbf{U}}{\partial \tau}+\mathbf{I}^M\frac{\partial \mathbf{U}}{\partial t}\right)_\Omega+\left(\phi,n_j\mathbf{F}^j+n_j\mathbf{G}^j\right)_{\partial\Omega} \\ &-\left(\frac{\partial}{\partial x}\phi,\mathbf{F}^j\right)_\Omega-\left(\frac{\partial}{\partial x}\phi,\mathbf{G}^j\right)_\Omega=0 \end{aligned} \qquad (4)$$

Integrate by parts again equation (4):

$$\begin{aligned} &\left(\phi,\frac{\partial \mathbf{U}}{\partial \tau}+\mathbf{I}^M\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial \mathbf{F}^j}{\partial x_j}+\frac{\partial \mathbf{G}^j}{\partial x_j}\right)=0 \\ &\Leftrightarrow\left(\phi,\frac{\partial \mathbf{U}}{\partial \tau}+\mathbf{I}^M\frac{\partial \mathbf{U}}{\partial t}\right)_\Omega+\left(\phi,\frac{\partial}{\partial x}\mathbf{F}^j\right)_\Omega+\left(\phi,\frac{\partial}{\partial x}\mathbf{G}^j\right)_\Omega \\ &+\left(\phi,n_j\left(\hat{\mathbf{F}}^j-\mathbf{F}^{j-}\right)+n_j\left(\hat{\mathbf{G}}^j-\mathbf{G}^{j-}\right)\right)_{\partial\Omega}+=0 \end{aligned} \qquad (5)$$

Where
$\hat{\mathbf{F}}^j{}_{\partial\Omega}=\hat{\mathbf{F}}^j\left(\hat{\mathbf{F}}^{j-},\hat{\mathbf{F}}^{j+}\right)\Big|_{\partial\Omega}$ and $\hat{\mathbf{G}}^j{}_{\partial\Omega}=\hat{\mathbf{G}}^j\left(\hat{\mathbf{G}}^{j-},\hat{\mathbf{G}}^{j+}\right)\Big|_{\partial\Omega}$
are the numerical fluxes.

Here $(.,.)$ represents the normal L_2 inner product and third term is flux vector. In this problem the numerical flux for convective terms is calculated by using the Lax-Friedrich flux and local discontinuous galerkin for viscous terms.

Here, we took the Kornwinder Dubiner function on straight sided triangle as the basis written in equation (5) (see figure 1 and 2):

$$\phi_{ij}(r,s)=\sqrt{\frac{2i+1}{2}}\sqrt{\frac{2i+2j+2}{2}}P_i^{0,0}\left(\frac{2(1+r)}{(1-s)}-1\right)P_j^{2=+1,0}(s) \qquad (6)$$

where, $P^{\alpha,\beta}$ is orthogonal Jacobi polynomial. All straight sided triangles are the image of this triangle under the map:

$$\begin{pmatrix} x \\ y \end{pmatrix}=-\left(\frac{r+s}{2}\right)\begin{pmatrix} v_x^1 \\ v_y^1 \end{pmatrix}+\left(\frac{1+r}{2}\right)\begin{pmatrix} v_x^2 \\ v_y^2 \end{pmatrix}+\left(\frac{1+s}{2}\right)\begin{pmatrix} v_x^3 \\ v_y^3 \end{pmatrix} \qquad (7)$$

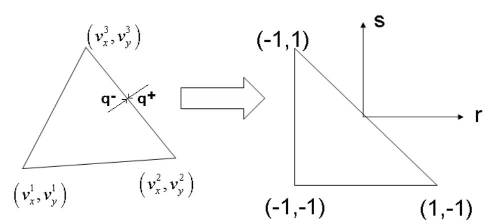

Figure 1: Coordinate transformation

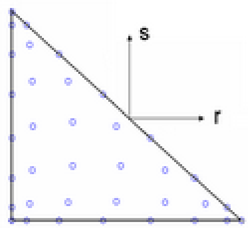

Figure 2: Seventh order Gauss Lobatto quadrature nodes

; equation (6),

$$p(r,s)=\sum_{i=0}^{N}\sum_{j=0}^{N-i}\phi_{ij}(r,s)\hat{p}_{ij} \quad (8)$$

$$p(r_n,s_n)=\sum_{m=1}^{m=M}\mathbf{V}_{nm}\hat{p}_m \quad (9)$$

$$\hat{p}_m=\sum_{m=1}^{m=M}\left(\mathbf{V}^{-1}\right)_{mj}p(r_j,s_j)$$

$$\frac{\partial p}{\partial r}(r,s)=\sum_{i=0}^{N}\sum_{j=0}^{N-i}\frac{\partial \phi_{ij}}{\partial r}(r,s)\hat{p}_{ij}=\hat{\mathrm{D}}^r\mathbf{V}^{-1}p(r,s)$$

$$\frac{\partial p}{\partial s}(r,s)=\sum_{i=0}^{N}\sum_{j=0}^{N-i}\frac{\partial \phi_{ij}}{\partial s}(r,s)\hat{p}_{ij}=\hat{\mathrm{D}}^s\mathbf{V}^{-1}p(r,s) \quad (10)$$

$$\hat{\mathbf{D}}^r=\frac{\partial\phi}{\partial r}\ ;\ \hat{\mathbf{D}}^s=\frac{\partial\phi}{\partial s}$$

where $\mathbf{V}_{ij}$ and N are Vandermonde matrix and the order of Jacobi polynomial respectively.

The semi discrete of equation (5) can be written in the following form:

$$\frac{\mathrm{d}\mathbf{U}}{\mathrm{d}\tau}+\mathbf{I}^M\frac{\mathrm{d}\mathbf{U}}{\mathrm{d}t}=\mathbf{R} \quad (11)$$

A dual time stepping approach is employed for marching equation (12) in time. The real time t is discretized using second order implicit backward difference formula. The resulting equation becomes:

$$\frac{\mathrm{d}\mathbf{U}^{n+1}}{\mathrm{d}\tau}+\mathbf{I}^M\frac{\left(3\mathbf{U}^{n+1}-4\mathbf{U}^n+\mathbf{U}^{n-1}\right)}{2\Delta t}=\mathbf{R}^{n+1} \quad (12)$$

where superscript n denotes the current time level t, and n-1 refers to the previous time step t-Δt, while the unknowns are calculated at time t+Δt by n+1. Equation (12) can be written in a simpler form as:

$$\frac{\mathrm{d}\mathbf{U}^{n+1}}{\mathrm{d}\tau}=\overline{\mathbf{R}}^{n+1} \quad (13)$$

where $\overline{\mathbf{R}}$ contains the right hand side of equation (12) and the second term on left hand side of this equation. Equation (13) represents a pseudo-time evolution of flow field and has no physical meaning until the steady state in pseudo-time is reached.

The Equation (8) is integrated in pseudo-time marching by using five stage of fourth order 2N-storage Runge-Kutta scheme as developed by [8]. The final equations are found as written in Eqs. (14) and (15) :

$$\frac{\mathrm{d}\mathbf{U}}{\mathrm{d}\tau}=\overline{\mathbf{R}}(\tau,\mathbf{U}(\tau)) \quad (14)$$

$$\begin{aligned} d\mathbf{U}_j &= A_j d\mathbf{U}_{j-1}+d\tau\overline{\mathbf{R}}(\mathbf{U}_j) \quad ; \quad j=1,5 \\ \mathbf{U}_j &= \mathbf{U}_{j-1}+B_j d\mathbf{U}_j \end{aligned} \quad (15)$$

where $d\tau$ is the pseudo-time step. The vectors **A** and **B** are the coefficients that will be used to determine the properties of the scheme.

4. INITIAL AND BOUNDARY CONDITIONS

Initial and boundary conditions The governing equations (1) or (3) require initial condition to start the calculation as well as boundary conditions at every time step. In the calculations presented in this paper, the uniform free-stream values are use as initial conditions: $p=p_\infty$; $u_1=u_{1\infty}$; $u_2=u_{2\infty}$. For external flow applications, the far-field bound is placed far away from the solid surface. Therefore, the free-stream values are imposed at the far-field boundary except along the outflow boundary where extrapolation for velocity components in combination with $p=p_\infty$ is used. On the solid surface, the no-slip condition is imposed for velocity components: $u_1=0$; $u_2=0$. The surface pressure distribution is determined by setting the normal gradient of pressure to be zero: $\frac{\partial p}{\partial n}=0$

5. RESULTS AND DISCUSSION

The accuracy of the proposed method is demonstrated by solving incompressible flow past 2-dimensional circular cylinder. The Reynolds number is varied from 20 to 40 for steady flow and from 100 to 200 for unsteady flow. The computational domain for steady flow is rectangle (-15, 25) × (-15, 15) and for unsteady flow is (-20, 20) × (-20, 80) wherein a circular cylinder of diameter d =1 placed at (0, 0). The mesh consists of 1228 triangles for steady flow and 5092 triangles for unsteady flow. For all the calculation, we took a

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fixed order of polynomial N =4, Prandtl number is 0.717 and fixed artificial compressibility parameter is equal to unity.

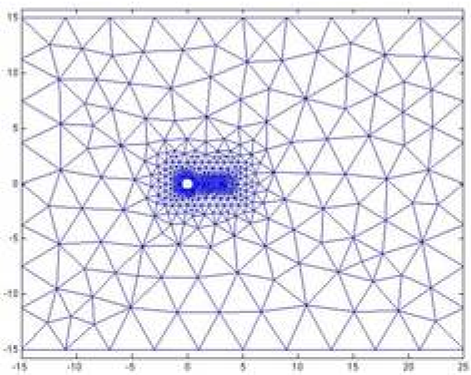

Figure 3a: Mesh for steady flow

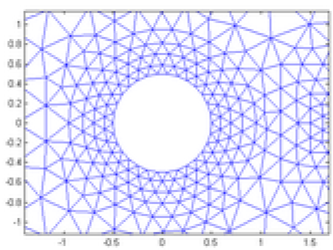

Figure 3b: Close-up mesh around cylinder for steady flow

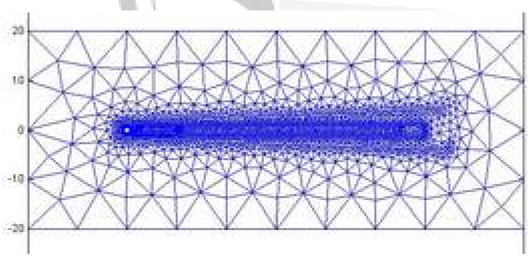

Figure 4a: Mesh for unsteady flow

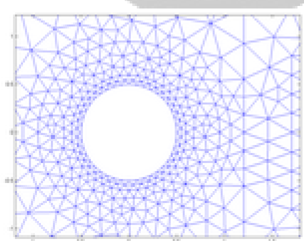

Figure 4b: Close-up mesh around cylinder for unsteady flow

Pseudo time step is $\Delta\tau = 0.0012$ for steady flow. The results can be seen in figure 5 and 6. In front of object, pressure strongly varies and a region of high pressure is formed near separation point and two regions of low pressure are developed next. From figure 5, it can be seen the development of a recirculation zone behind the object with reverse velocity. Figure 5 and 6 give the computed pressure for different values of Reynolds number (Re=20 and 40). As shown, the DG scheme gives excellent pressure stabilization, with the computed pressure contours being highly smooth and non-oscillatory.

The calculated results for steady flows are compared with the other numerical data. Table 1 compares the Drag coefficient (Cd). The agreement is quite good.

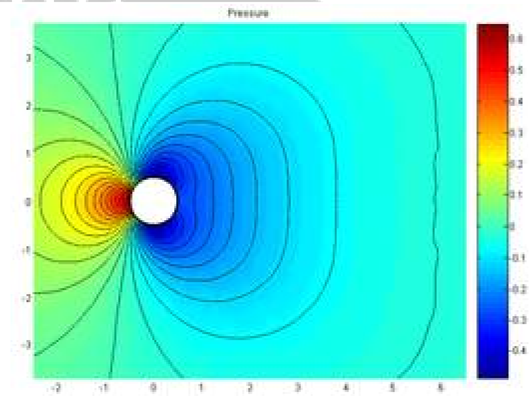

Figure 5a: Isobar of Re=20

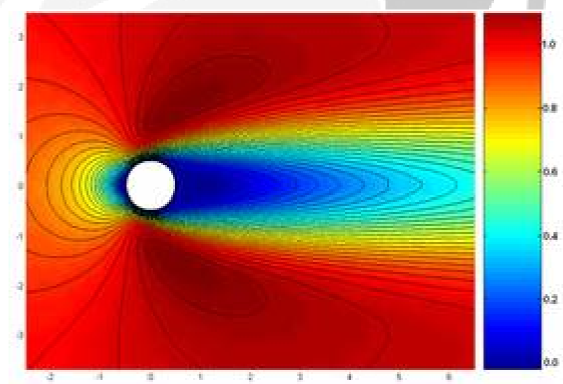

Figure 5b: Horizontal velocity of Re=20

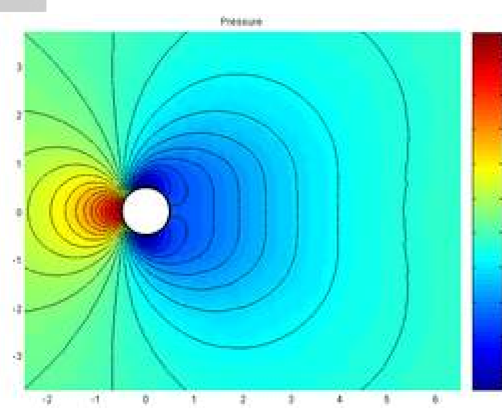

Fig. 6a. Isobar of Re=20

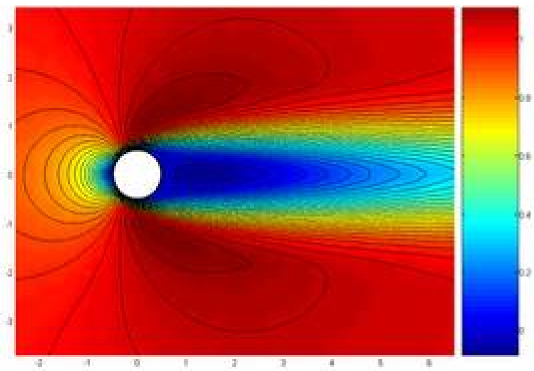

Figure 6b: Horizontal velocity of Re=20

Table 1. Drag coefficient (Cd) comparisons for steady flows

Author	Re=20	Re=40
Takami *et al.**	2.003	**1.536**
Dennis *et al.**	2.045	**1.522**
Tuann *et al.**	2.253	**1.675**
Ding *et al.**	2.180	**1.713**
Nithiarasu *et al.**	2.060	**1.564**
Thomas*	2.076	**1.603**
Present work	**2.040**	**1.527**

*: adapted from [9]

The ability of the discontinuous galerkin scheme with dual time-stepping to simulate transient flow is illustrated here by computing the vortex shedding in the wake of flow past a circular cylinder at Re = 100 and 200. This has been a popular test case for validating the transient part of numerical schemes. The problem is solved using is Δt = 0.1 and $\Delta\tau$ = 0.001 and total of 250 pseudo-time iterations. Figure 7 provides a full simulation analysis of the Cl and Cd histories for each real time step on the mesh. As can be seen, once the initial transient stage has passed, the simulation settles down to an almost periodic convergence pattern. After the initial transient, each variable develops a periodic variation. This is due to the periodic shedding of vortices from behind the cylinder. This can be seen more clearly in figure 7, 8 and 9. A quantitative analysis of the results was also conducted and is shown Table 2. Generally, all the results shown for DG scheme are in good agreement with the other results.

Table 2: Cl, Cd and St comparisons for unsteady flows

Author	Re=100			Re=200		
	Cl	Cd	St	Cl	Cd	St
Roger & Kwak [6]	±0.358	1.376 ±0.011	**0.163**	**±0.65**	1.23±0.05	**0.185**
Pontaza [10]	±0.356	1.356	**0.167**	-	-	-
Mittal [11]	±0.356	1.386	**0.169**	-	-	-
Roshko (exp) [12]	-	-	-	-	-	**0.18**
Bintoro & Pranowo [13]	-	-	-	**±0.552**	1.24±0.09	**0.17**
Rosenfeld [12]	-	-	-	±0.674	1.329±0.044	**0.197**
Li [14]	-	-	-	±1	1.17±0.15	**0.18**
Present work	**±0.3644**	**1.3440 ±0.013**	**0.1563**	**±0.645**	**1.311± 0.036**	**0.176**

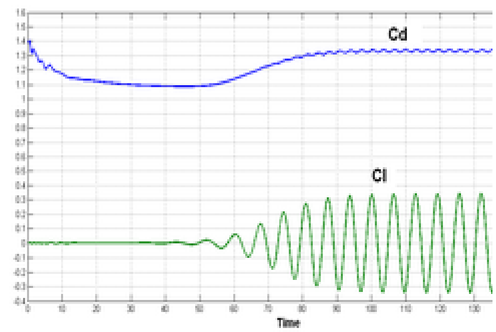

Figure 7a: Cd & Cl History for Re=100

The qualitative results are shown in figures 8 and 9. Here, the contours of pressure and vorticity are shown, for the real non-dimensional times of 100 and 120 respectively. All results are of high quality with no non-physical oscillations. The narrowing of the wake and the increase in shedding frequency, as the Reynolds number increases, is clear from these plots. We observe that at higher Reynolds numbers the vortices in the far-downstream wake coalesce and the region of coalescence moves upstream as the Reynolds number increases.

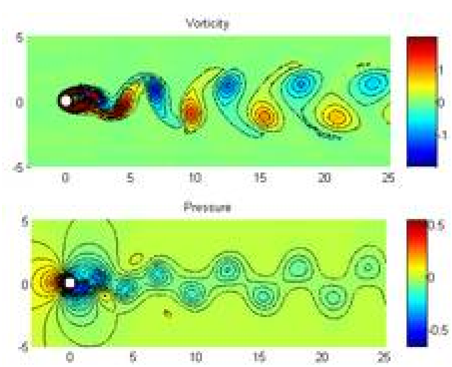

Figure 8a: Iso-vorticity and Isobar for Re=100 at *t*=100

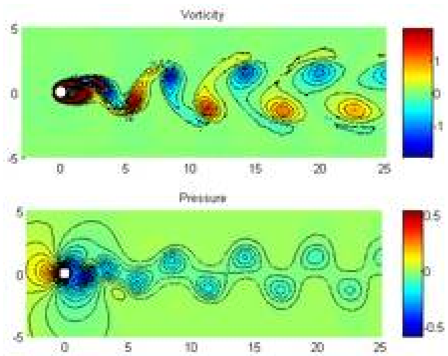

Figure 8b: Iso-vorticity and Isobar for Re=100 at *t*=120

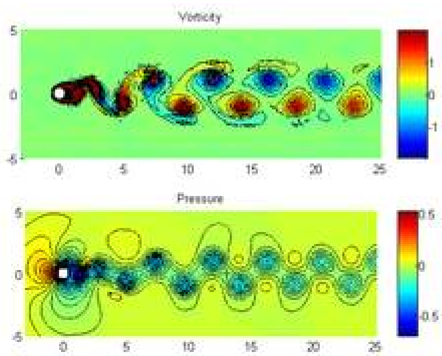

Figure 9a: Iso-vorticity and Isobar for Re=200 at *t*=100

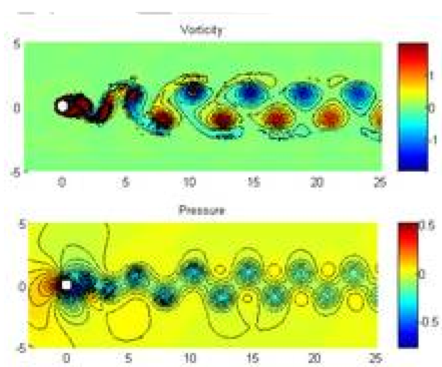

Figure 9a: Iso-vorticity and Isobar for Re=200 at *t*=120

It can be seen from the plots that the proposed method is stable for long time simulations. In addition, the plots show evidence that the outflow boundary condition allows for a smooth exit of the flow field and does not distort the flow upstream.

7. CONCLUSION

In this paper, we have presented a discontinuous galerkin method for steady and unsteady in artificial compressibility formulation connection with a dual time stepping approach. The method exhibits good numerical stability. The numerical results have a good agreement with the proposed experimental and numerical results reported in the previous studies.

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