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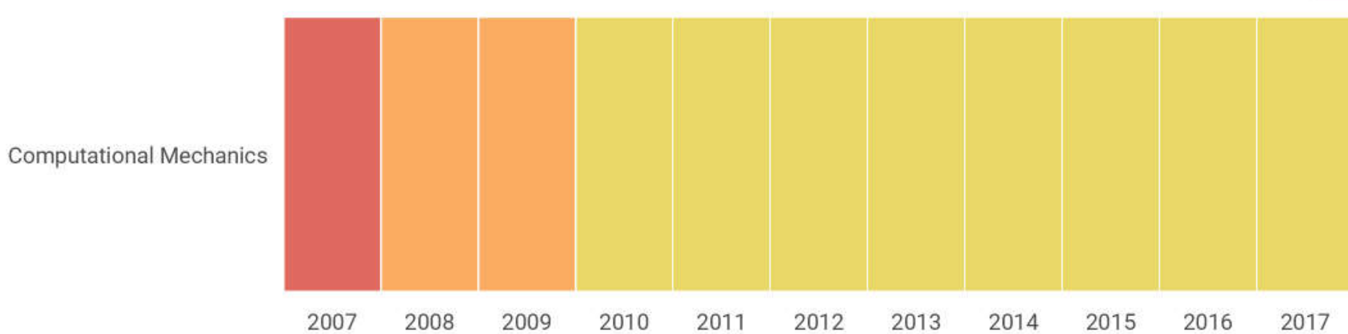

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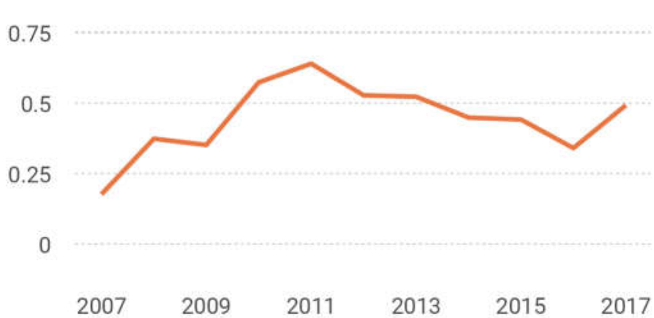

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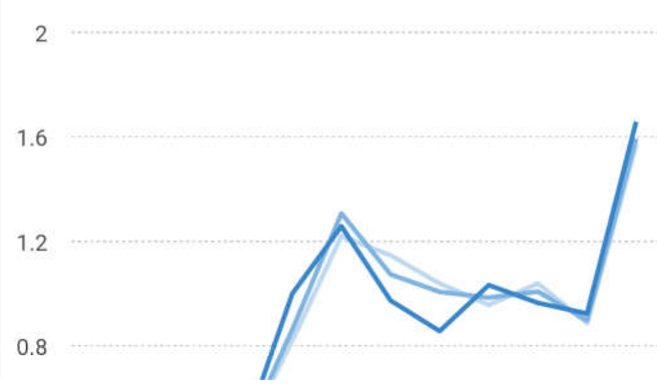

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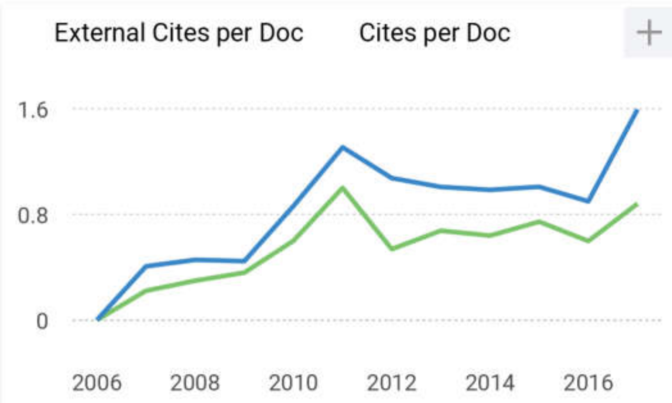

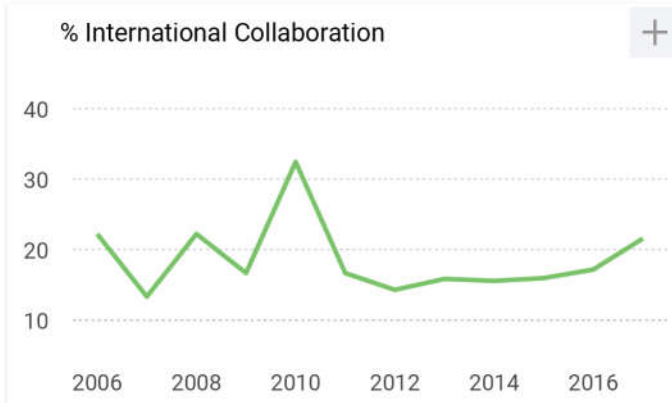

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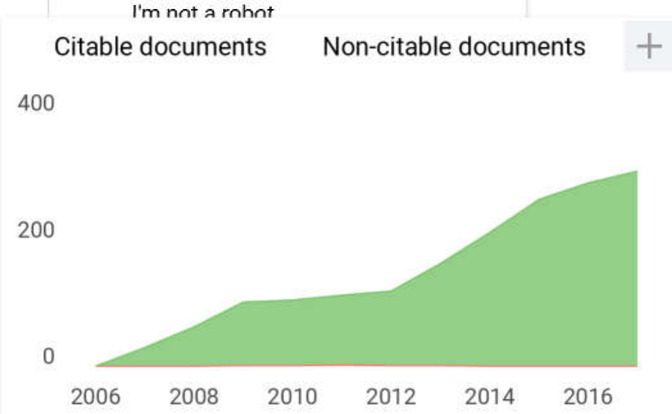

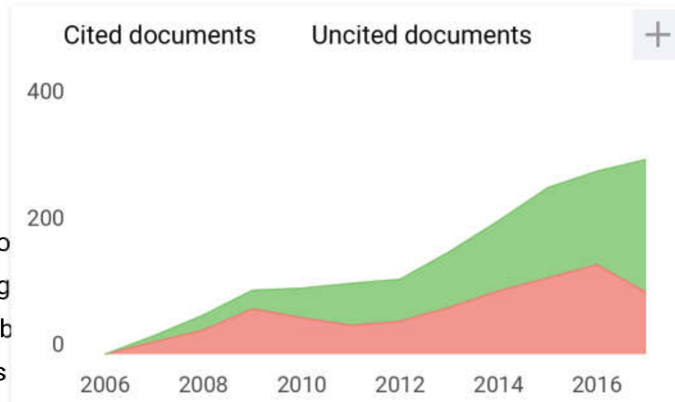

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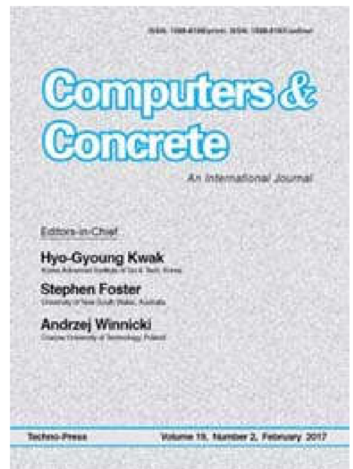

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Nonlinear finite element analysis of torsional R/C hybrid deep T-beam with opening

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Abstract. A nonlinear finite element analysis of R/C hybrid deep T-beam with web opening subjected to pure torsion is presented. Hexahedral 8-nodes and space truss element were used for modeling concrete and reinforcement. The reinforcement was assumed perfectly bonded to the corresponding nodes of the concrete element. The constitutive relations for concrete and reinforcement are based on the modified field theory and elastic perfectly plastic. The smear crack approach was adopted for modeling the crack. The torque-twist angle relationship curve based on the finite element analysis was compared to the experimental results. The comparison shows that the curve of torque-twist angle predicted by the nonlinear finite element analysis is linear before cracking and close to the experimental result. After cracking, the curve becomes nonlinear and stiffer compared to the experimental result.

Keywords: hybrid deep T-beam; web opening; pure torsion; finite element analysis; smear crack

1. Introduction

In order to passage ducting or plumbing and to safe head room in high rise building, it is often required an opening in the beam. Beams with span-depth ratio equal to or less than four are categorized as deep beam (ACI Building Code 2008). Deep beam is a structural element where the shear forces are dominant and play a major role in safety assessment (Pimentel *et al.* 2008). Although deep beam is rarely subjected to pure torsion, it is beneficial to do a research into the behavior of the deep beam subjected to pure torsion for the basis of future study under combine loading.

Reducing the mass of building is an alternative choice in order to minimize the effect of earthquake on the building. Substituting heavier normal weight concrete of the floor of building with light weight concrete can be chosen as an alternative for minimizing the earthquake effect. Generally, it is a slab or floor cast on the top of the beams. Therefore, it is reasonable to take into account the T-shape section of such beam in evaluation of torsional behavior of the beam.

Experimental programs in torsional behavior of reinforced concrete (R/C) hybrid deep T-beams were conducted by Lisantono *et al.* (2001, 2002a, 2002b, 2004). In these experimental programs, the flange and the web of the deep T-beams cast of light weight concrete and normal weight

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concrete. Generally, the torsional behavior of R/C hybrid deep T-beams with web opening derived based on the experimental programs are essentially linear before cracking, after which the torque increased nonlinearly with increasing the twist angle. Upon reaching their ultimate torque strength in the beams decreased nonlinearly and concluded with a yield condition prior to collapse.

Some researchers proposed theory for predicting the torsion strength of reinforced concrete members such as Softened Truss Model (Hsu and Mo 1985a, 1985b, Hsu 1988, 1991a, 1991b); Softened Membrane Model for Torsion or SMMT (Jeng and Hsu 2009); combination of the Variable Angle Truss Model with Modified Compression Field Theory (Valipour and Foster 2010) and step by step simplification of the Softened Membrane Model for Torsion (Jeng 2010). Although through complicated iterative solution, the proposed theories can predict the entire torque-twist curve except the step by step simplification of the SMMT which gives a set simple formula for direct calculation of cracking torque and twist. While some researchers proposed the analytical theory for predicting torsional strength of the beams, others also tried to use finite element analysis to study the behavior of structures (Mahmood 2007, Shayanfar and Safiey 2008, Alshoaibi 2010, Aurich *et al.* 2011), because finite element analysis has become an important tool (Broo *et al.* 2008). Mahmood (2007) conducted a nonlinear finite element analysis for reinforced concrete beam under pure torsion, however this research is limited only for shallow beam. Since research in nonlinear finite element analysis of deep beams has less attention, therefore study of nonlinear finite element analysis of deep beam is necessary.

2. Finite element modeling

2.1 Crack approach

Kotsovos and Pavlovic (1995) stated that there are two basic schemes alternatives for crack modeling, the first is discrete-crack approach and the second is smeared-crack approach. The discrete-crack approach introduces an actual gap in the finite element mesh at the location of the crack. It can be achieved by doubling and separating the nodal coordinates lying along individual crack paths. This implies changes in the numbering of nodes and element connectivities which in turn affects the global stiffness matrix. While the smeared-crack approach is easier than discrete-crack approach because the smeared models simply replace the uncracked material stiffness matrices (D-matrices) by the cracked one. In this research, the smeared-crack approach was adopted for modeling the cracked element in finite element analysis.

Kotsovos and Pavlovic (1995) also stated that uncracked D-matrices of concrete is isotropic before cracking and orthotropic after cracking. The D-matrices after cracking for 3-dimensions cases can be written as follows

$$[D]=\begin{bmatrix} (2G+\mu) & \mu & 0 & 0 & 0 & 0 \\ \mu & (2G+\mu) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta G & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta G \end{bmatrix} \tag{1}$$

where

$$G = \frac{E}{2(1+\upsilon)} \tag{2}$$

$$\mu = \frac{\upsilon E}{(1+\upsilon)(1-2\upsilon)} \tag{3}$$

υ = Poisson's ratio
β = Shear Retention Factor (SRF) taken as 0.5 for deep beam (Lisantono 2005)

2.2 Constitutive model of material

Nonlinearity of finite element analysis can be performed by reflecting the nonlinearity of material, structural geometric, contact and boundary condition. Modified Compression Field Theory (MCFT) proposed by Vecchio and Collins (1986) is adopted for the nonlinearity of the concrete in this research. The stress-strain formulation of MCFT has been applied to a variety of concrete structural analysis problems (Hsu and Mo 1985a, 1985b, Mau and Hsu 1987, Vecchio 1989). The stress-strain relationships of concrete according to MCFT are depicted in Figs. 1 and 2 for concrete in principle tensile direction and principal compression direction, respectively.

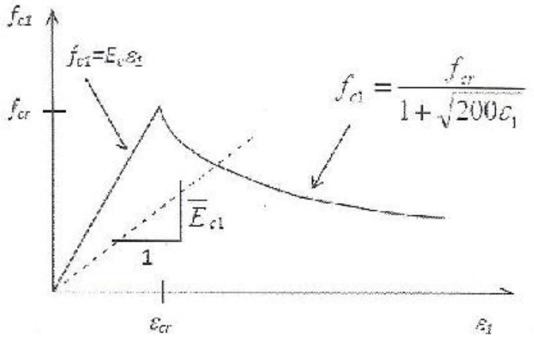

Fig. 1 Concrete in principal tensile direction

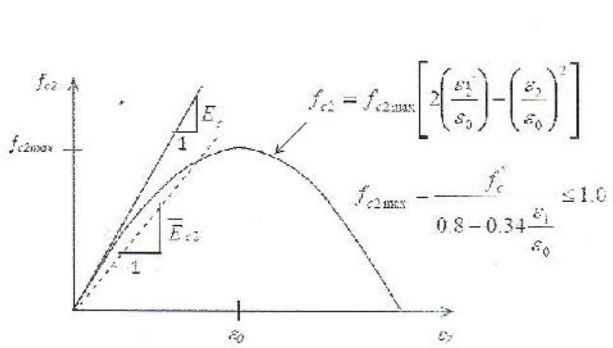

Fig. 2 Concrete in principal compression direction

For concrete in tension, prior to cracking, a linear stress-strain relationship is used. Thus the principal tensile stress is

$$f_{c1} = E_c \varepsilon_1 \tag{4}$$

After cracking, the principal tensile stress is

$$f_{c1} = \frac{f_{cr}}{1+\sqrt{200\varepsilon_1}} \tag{5}$$

Eq. (5) is reflecting the tension stiffening effect.
For concrete in compression, the principal compression stress is

$$f_{c2} = f_{c2\max}\left[2\left(\frac{\varepsilon_2}{\varepsilon_0}\right)-\left(\frac{\varepsilon_2}{\varepsilon_0}\right)^2\right] \tag{6}$$

where

$$f_{c2\max} = \frac{f_c'}{0.8-0.34\dfrac{\varepsilon_1}{\varepsilon_0}} \leq 1.0 \tag{7}$$

According to Vecchio (1989) and Kwak and Filipou (1990) that concrete after cracking can be assumed as orthotropic material, hence the influence of Poisson's ratio can be neglected. For the 3-D cases, the D-matrices for uncracked concrete may be written as

$$[D] = \begin{bmatrix} (2G+\mu) & \mu & \mu & 0 & 0 & 0 \\ \mu & (2G+\mu) & \mu & 0 & 0 & 0 \\ \mu & \mu & (2G+\mu) & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix} \tag{8}$$

According to Kotsovos and Pavlovic (1995) when the state of stress at a Gauss point reaches the triaxial envelope involving at least one principal tensile component for the first time, a crack plane is assumed to form in the direction orthogonal to the maximum principal tensile stress. Therefore the D-matrices after cracking becomes

$$[D] = \begin{bmatrix} (2G+\mu) & \mu & 0 & 0 & 0 & 0 \\ \mu & (2G+\mu) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix} \tag{9}$$

If it is considered the shear retention factor, Eq. (9) exactly becomes the same as Eq. (1).

According to Vecchio (1989) and Kwak and Filipou (1990), concrete after cracking can be assumed as orthotropic material, hence the influence of Poisson's ratio can be neglected. When the Poisson's ratio is neglected, Eq. (1) becomes

$$[D]_c = \begin{bmatrix} \overline{E}_{c1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \overline{E}_{c2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{G}_c & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta\overline{G}_c & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta\overline{G}_c \end{bmatrix} \tag{10}$$

where

$$\overline{G}_c = \frac{\left(\overline{E}_{c1}\overline{E}_{c2}\right)}{\left(\overline{E}_{c1} + \overline{E}_{c2}\right)} \tag{11}$$

and where $\overline{E}_{c1}$ and $\overline{E}_{c2}$ are the secant moduli and stated in the following equation

$$\overline{E}_{c1} = \frac{f_{c1}}{\varepsilon_1} \tag{12}$$

$$\overline{E}_{c2} = \frac{f_{c2}}{\varepsilon_2} \tag{13}$$

where,

f_{c1} = the principle tensile stress

f_{c2} = the principle compressive stress

ε_1 = the principle tensile strain

ε_2 = the principle compressive strain

Stresses in the reinforcement are using the elastic-perfectly plastic, as shown in Fig. 3.

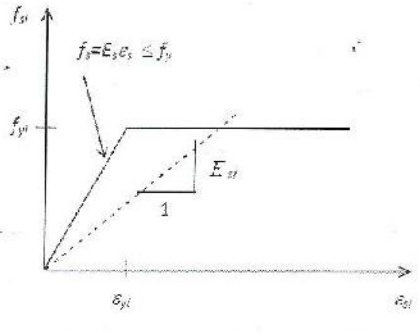

Fig. 3 Constitutive relation of reinforcement

Thus, the stresses of reinforcement for 3-dimension cases are shown in the following the equation,

$$f_{sx} = E_s \varepsilon_x \leq f_{yx} \tag{14}$$

$$f_{sy} = E_s \varepsilon_y \leq f_{yy} \tag{15}$$

$$f_{sz} = E_s \varepsilon_z \leq f_{yz} \tag{16}$$

where,
f_{sx}, f_{sy}, f_{sz} = the stresses of reinforcement in x, y and z direction, respectively.
E_s = the elastic modulus of reinforcement.

2.3 Element modeling

Hexahedral 8-nodes element is used for modeling finite element analysis. The reinforcement is assumed perfectly bonded to the corresponding nodes of concrete element. Finite Element Analysis Program (FEAP free version) was used for nonlinear finite element analysis of the R/C hybrid deep T-beams. Because the constitutive model available in the FEAP is elastic isothropic, it needs to develop subroutines for modeling constitutive material model in the FEAP. Lisantono (2004) developed subroutines for nonlinear finite element analysis of R/C hybrid deep T-beams. There were two subroutines to be developed, the first subroutine as an interface of materials model and the second as constitutive model of materials. The program was implemented to analyze four R/C deep T-beams, namely B4HS, B4HOD1, B4HO and B4HOD3 as depicted in Figs. 4-7, respectively.

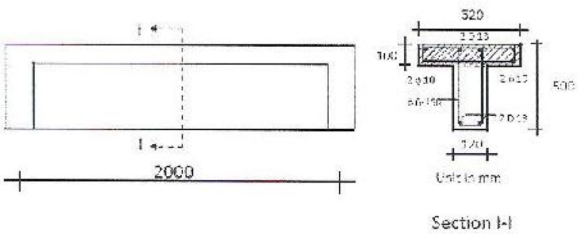

Fig. 4 Specimen of B4HS

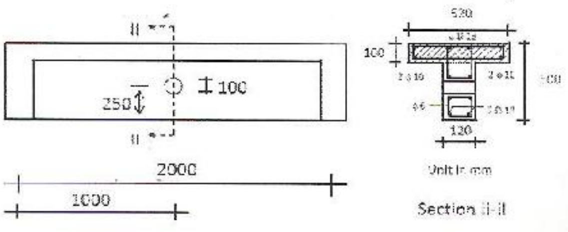

Fig. 5 Specimen of B4HOD1

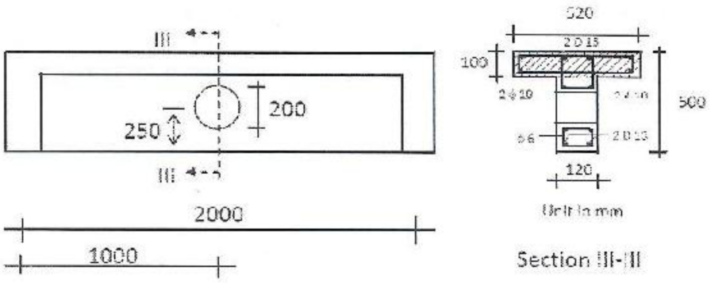

Fig. 6 Specimen of B4HO

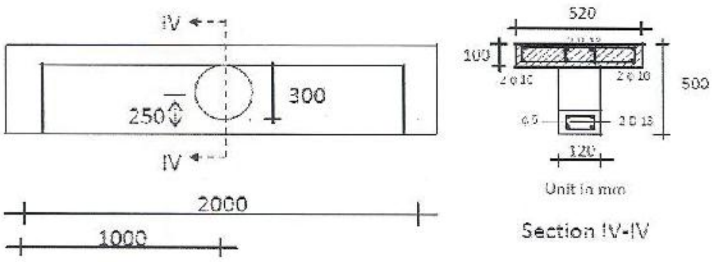

Fig. 7 Specimen of B4HOD3

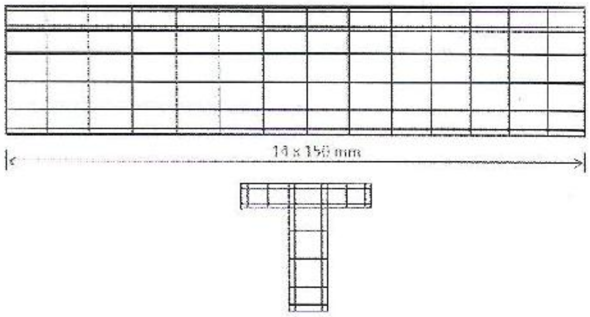

Fig. 8 Meshing of B4HS

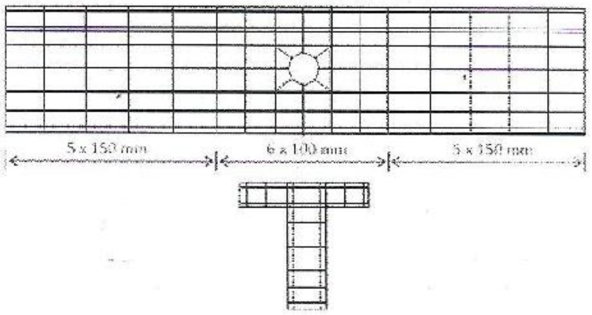

Fig. 9 Meshing of B4HOD1

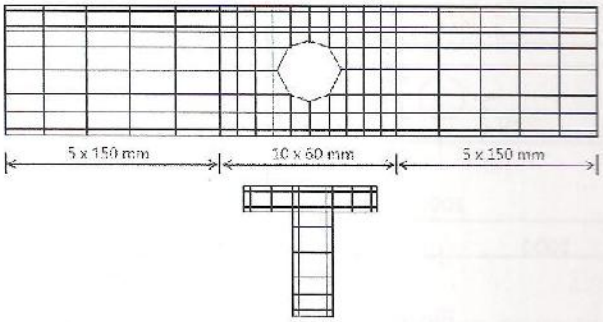

Fig. 10 Meshing of B4HO

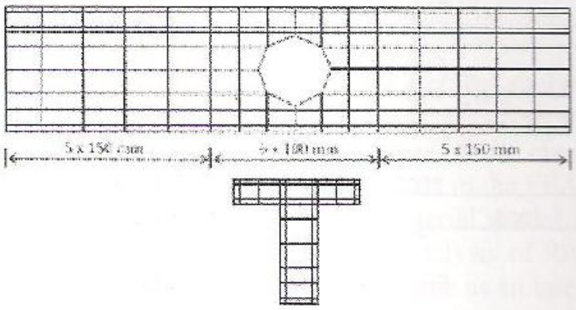

Fig. 11 Meshing of B4HOD3

Table 1 The properties of light weight concrete at 28 days

Compressive strength (MPa)	Tensile strength (MPa)	Modulus elasticity (MPa)	Poisson ratio	Density of concrete (Kg/m^3)
40.74	2.79	14698.33	0.221	1721.98

Table 2 The properties of normal weight concrete at 28 days

Compressive strength (MPa)	Tensile strength (MPa)	Modulus elasticity (MPa)	Poisson ratio	Density of concrete (Kg/m^3)
35.40	3.32	25149.67	0.205	2356.96

The finite element meshing of the beams are shown in the Figs. 8-11. As stated by Cervenka *et al.* (2005), the spurious mesh sensitivity caused by strain softening due to smeared tensile cracking can be remedied by crack band model. The mesh sensitivity caused by strain softening due to smeared tensile cracking is ignored in this paper.

2.4 Material data

The flange of the deep T-beams was cast of light weight concrete and the web was cast of normal weight concrete. The material properties of the light weight concrete and the normal weight of concrete can be seen in Tables 1 and 2.

The yield strength of the steel bars with diameter of 13 mm and 10 mm was 350 MPa, while the yield strength of the steel bar with diameter of 6 mm was 260 MPa. The modulus elasticity of the steel was 200,000 MPa.

3. Results and discussion

Comparison the torque-twist curve between the finite element method (FEM) and experimental results of the beam B4HS, B4HOD1, B4HO and B4HOD3 can be seen in the Figs. 12-15, respectively.

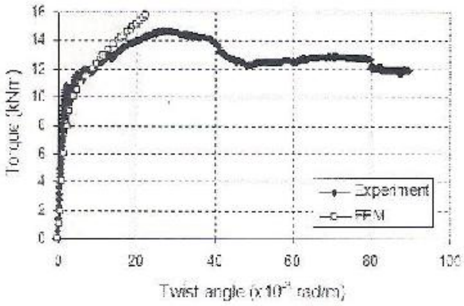

Fig. 12 Comparison torque-twist curve of B4HS

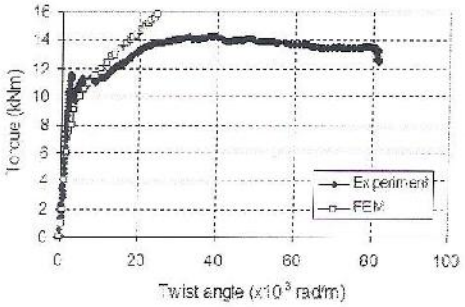

Fig. 13 Comparison torque-twist curve of B4HOD1

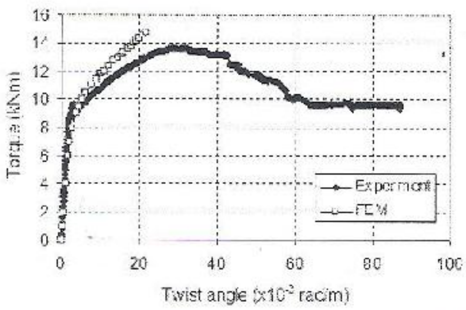

Fig. 14 Comparison torque-twist angle of B4HO

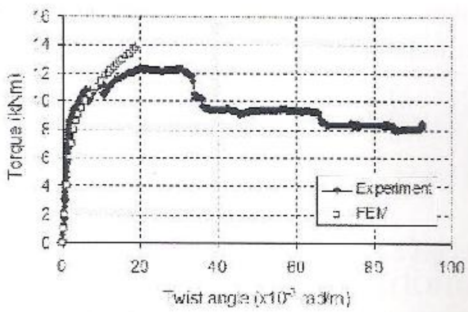

Fig. 15 Comparison torque-twist angle of B4HOD3

Figs. 12-15 show that generally the torque-twist angle curve of FEM results is linear before cracking, and becomes nonlinear after cracking. Before cracking when the curve is linear until some stages of nonlinear, up to this stage the curve of FEM results close to the experimental results. However, after cracking the torque of FEM result is increasing nonlinearly with very small increasing of the twist angle that makes the torque-twist curve of FEM becomes stiffer compared to the experimental result.

The stiffer of FEM result is due to the perfect bonding assumption between concrete and steel bar which gives smaller twist angle compared to experimental results. This stiffer curve is also similar to the research done by Fafitis and Won (1994). Fafitis and Won (1994) conducted nonlinear finite element analysis of deep beams subjected to bending and shear, the results showed that the finite element analysis results also stiffer than the experimental results. They stated also that the stiffer was also due to the perfect bonding assumption between concrete and steel bar. Therefore, for the next investigation in nonlinear finite element analysis of the deep beams subjected to pure torsion needs to take into account the influence of bond-slip because the mechanism of bond-slip is significantly affect the behavior of element structure especially in crack propagation, crack width, and ductility (Lundgren 1999). It was also mentioned by Shayanfar and Safiey (2008) that one of the most effective parameters in new approach for nonlinear finite element analysis must be taking into consideration is bond-slip behavior.

4. Conclusions

Applying the nonlinear finite element analysis on the torsional R/C hybrid deep T-beams without or with web opening, several conclusions are obtained.

1. Generally the torque-twist curve behavior of the deep beams based on FEM procedures is linear before cracking and increasing nonlinearly after cracking with very small increasing of the twist angle.

2. The torque-twist curve of the deep beams based on FEM procedures is still stiffer than that of experimental results. The stiffer result is caused by the perfect bonding between concrete and still bars. Hence, it still needs to take into account the bonding mechanism in FEM procedure.

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