CHAPTER 2

MODELLING THE BRIDGE

2.1. Finite Element Model of the Bridge

Based on the detailed information of the Cape Girardeau bridge, a three-dimensional finite element model was developed in MATLAB® (1997) by Dyke et al (2002). The evaluation model used in this benchmark study is linear. However, the stiffness matrices used in this linear model are those of the structure determined through a nonlinear static analysis corresponding to the deformed state of the bridge with dead loads (Wilson and Gravelle, 1991). The nonlinear static analysis is performed using ABAQUS® (1998), where the element mass and stiffness matrices are output to MATLAB® for assembly.

The finite element model employs beam elements, cable elements and rigid links. This model has a large number of degree of freedom and high frequency dynamics. Some assumptions were made regarding the behavior of the bridge while retaining the fundamental behavior of the bridge. The constraints are applied to the mass and stiffness matrices, and a reduction is performed to reduce the size of the model to a more manageable model. The first ten frequencies of the evaluation model are 0.2899, 0.3699, 0.4683, 0.5158, 0.5812, 0.6490, 0.6687, 0.6970, 0.7102, and 0.7203 Hz. These steps are summarized in Fig. 2.1.

This model is used as a basis of comparison for the controlled system in which the deck-tower connections are fixed (the dynamically stiff shock transmission devices are present).

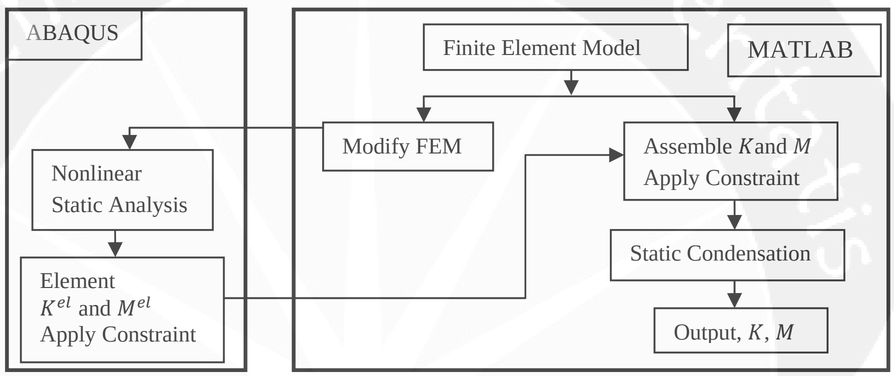

Fig 2.1. Creating the Evaluation model (Dyke et al, 2002)

To make it possible for designers/researchers to place devices acting longitudinally between the deck and the tower, a modified evaluation model is formed in which the connections between the tower and the deck are disconnected. If a designers/researcher specifies devices at these nodes, the second model will be formed as the evaluation model, and the control devices should connect the deck to the tower. As one would expect, the frequencies of this model are much lower than those of the nominal bridge model. The first ten frequencies of this second model are 0.1618, 0.2666, 0.3723, 0.4545, 0.5015, 0.5650, 0.6187, 0.6486, 0.6965, and 0.7094 Hz.

2.2. Reduced Model of the Bridge of Cape Girardeau

The reduction is done through static condensation technique. The active DOFs retained in the model include (Dyke et al., 2002):

- the nodes at the top at each tower,
- the lowest nodes at which cables are connected on each tower,
- nodes at the joints of the tower,
- nodes or DOFs of elements whose shear and overturning moment are among the design criteria,
- approximately every third mode of the bridge deck,
- rotational DOFs about the longitudinal and vertical axis of all spinal decks nodes.

These locations are indicated in Fig 2.2. The result is a model with 419 degree of freedom.

Static condensation is performed by Dyke et al., by partitioning the mass and stiffness matrices into active and dependent DOF as in

$$M = \begin{bmatrix} M_{aa} & M_{ad} \\ M_{da} & M_{dd} \end{bmatrix}, \quad K = \begin{bmatrix} K_{aa} & K_{ad} \\ K_{da} & K_{dd} \end{bmatrix} \tag{2-1}$$

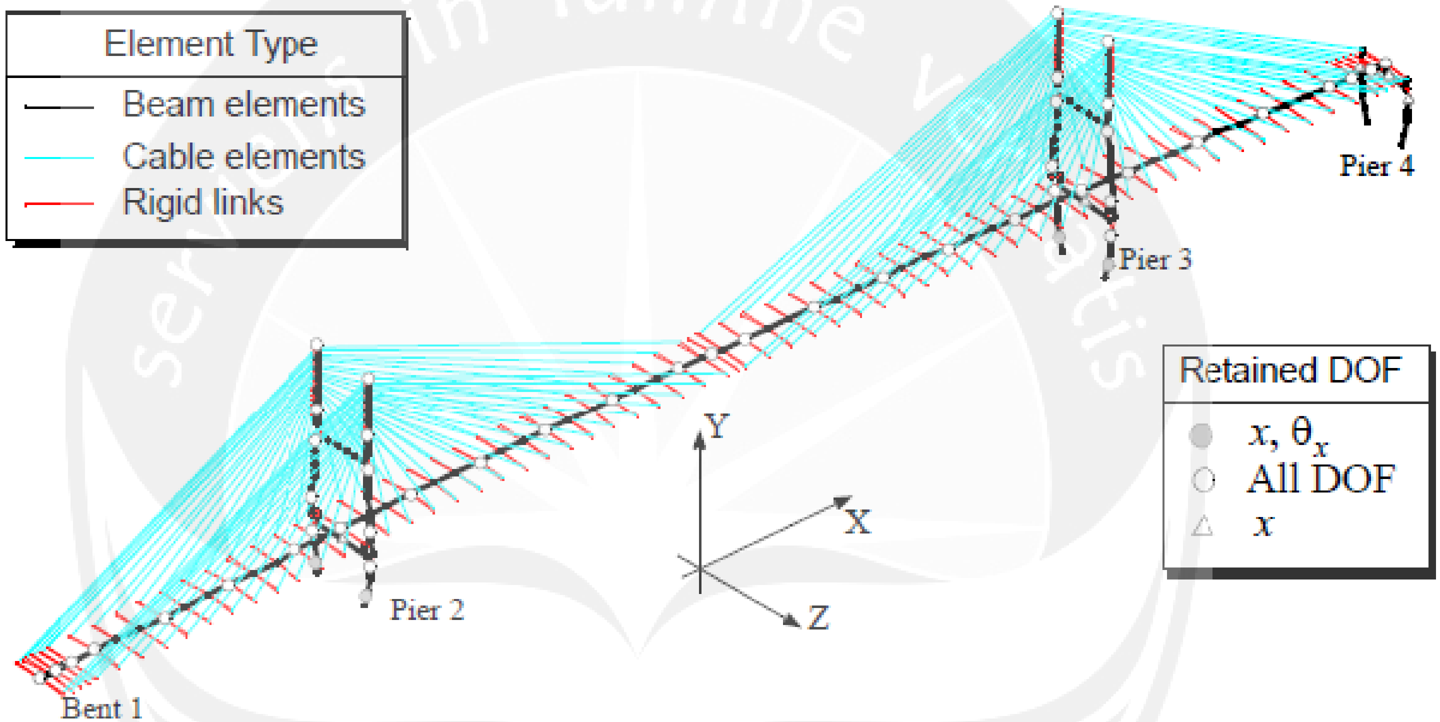

Fig 2.2. Elements and DOF retained for the Cape Girardeau Bridge (Dyke et al, 2002)

where M is the mass matrice, K is the stiffness matrice, a denote the active DOF, and d for the dependent DOF. Assuming that no loads are applied to the dependent DOFs, the system equation for static equilibrium is written as

$$\begin{bmatrix} K_{aa} & K_{ad} \\ K_{da} & K_{dd} \end{bmatrix} \begin{bmatrix} \widehat{U} \\ \overline{U} \end{bmatrix} = \begin{bmatrix} P_a \\ 0 \end{bmatrix} \tag{2-2}$$

where $\widehat{U}$ is the active, and $\overline{U}$ is the dependent displacement vector. Using the second row of eq.(2-2), the transformation matrix is obtained as

$$T_R = \begin{bmatrix} I \\ -K_{dd}^{-1} K_{da} \end{bmatrix} \tag{2-3}$$

where $T_R$is the static transformation matrix, and I is the identity matrix of appropriate size, such that

$$\begin{bmatrix} \widehat{U} \\ \overline{U} \end{bmatrix} = T_R \widehat{U} \tag{2-4}$$

The transformed mass and stiffness matrices are then as follows

$$\widehat{M} = T_R^T M T_R \;\text{ and }\; \widehat{K} = T_R^T K T_R \tag{2-5}$$

The corresponding coefficient matrices for the ground excitation and the control forces are given by

$$\widehat{\Gamma} = T_R^T M T_R \;\text{ and }\; \widehat{\Lambda} = T_R^T \Lambda \tag{2-6}$$

Prior to making this transformation, Γ and Λ must be reordered into active and dependent degrees-of-freedom. Application of this reduction scheme to the full model of the bridge resulted in a 419 DOF reduced order model. The first 100 natural frequencies of the reduced model (up to 3.5 Hz) are in good agreement with those of the 909 DOF structure. The damping in the system is defined based on the assumption of modal damping. The damping matrix was developed by assigning 3% of critical damping to each mode. This value was selected to be consistent with assumptions made during the design of the bridge. The reduced system was used to construct the damping matrix using

$$\hat{C} = \widehat{M}\Phi\begin{bmatrix} 2\zeta_1\omega_1 & 0 & 0 \\ 0 & \ldots & 0 \\ 0 & 0 & 2\zeta_n\omega_n \end{bmatrix} \tag{2-7}$$

where Φ is the modal matrix, and ζ_1 and ω_1 are the natural frequency [rad/sec] and modal damping ratio of the ith mode, respectively. The resulting equation of motion for the damped structural system is

$$\widehat{M}\ddot{\widehat{U}} + \hat{C}\dot{\widehat{U}} + \widehat{K}\widehat{U} = -\hat{\Gamma}\ddot{x}_g + \hat{\Lambda}f \tag{2-8}$$

where $\widehat{U}$ is the displacement vector of active DOFs. This model is termed the evaluation model. It is considered to portray the actual dynamics of the bridge and will be used to evaluate various control systems.

Note that this model always includes the effects of the shock transmission devices, which constraint longitudinal motion. The evaluation model and earthquake inputs are fixed for this benchmark problem.