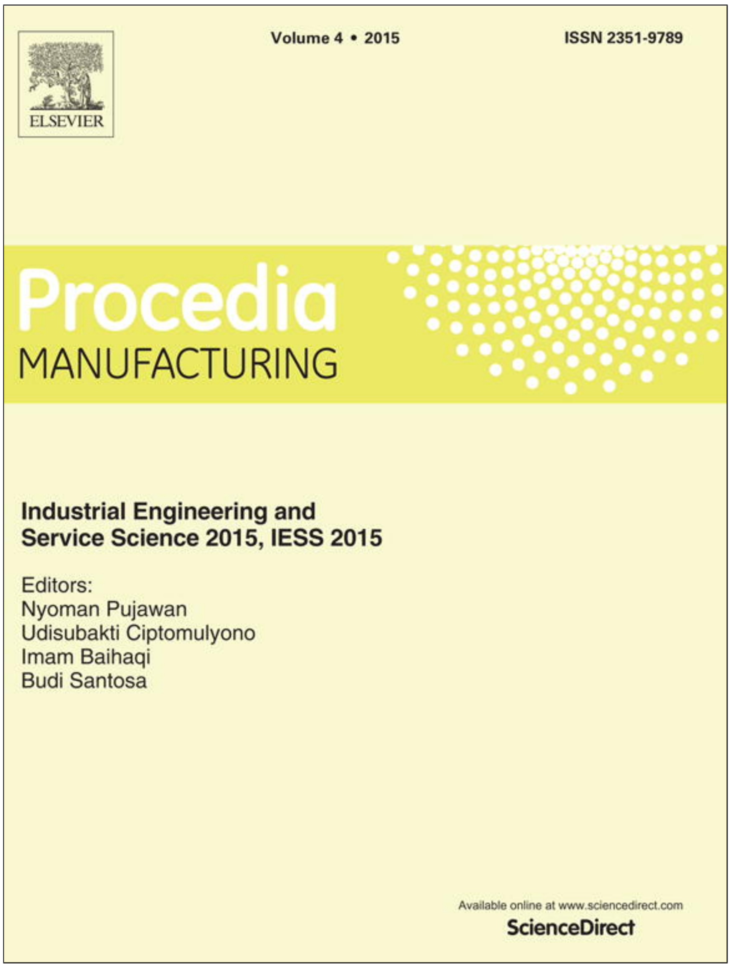
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# Economic lot scheduling problem with two imperfect key modules

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**Abstract**

This paper considers an Economic Lot Scheduling Problem (ELSP) with two imperfect Key Modules (KMs), in which extending similar work on the Economic Production Quantity scope. It is assumed that each KM has its own probability to shift from in-control state to out-of-control state. When the production shifts to out-of-control state, it starts to produce defected items. The problem in this paper is defined as finding the cycle times for several items under ELSP with two KMs context in order to minimize the total cost covering holding cost, setup cost and quality-related cost. A series of modelling was done in order to develop the formula and algorithms to solve the cycle time T under the Independent Solution (IS) and Common Cycle (CC) approach. A numerical illustration is given based on the modified Bomberger (1966) stamping problem.


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*Keywords:* Economic lot scheduling problem; imperfect production system; two key modules; lot sizing

## 1. Introduction

Economic Lot Scheduling Problem (ELSP) is a problem of scheduling production of several different items over the same facility on a repetitive basis [3]. A particular case of ELSP for imperfect production process where the items produced may be of imperfect quality has been discussed [2]. Another research under such theme by emphasizing on imperfect production system and setup times has also been conducted [9]. Under imperfect production process, it is assumed that the production starts in in-control state. After a certain elapsed time, the production system may shift from in-control to out-of-control state and may start to produce defected items until

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the setup of the next production. A certain number of defected items affect the quality-related cost and thus may increase the total cost. To solve this problem under ELSP context, the model to calculate the cycle times of multiple items being produced in the system in order to minimize the total cost covering setup cost, holding cost and quality-related cost have been provided [2, 9].

Among the papers being reviewed under ELSP theme, no papers provided the model to solve ELSP with two imperfect key modules. Our model considers a production system with two imperfect key modules under the ELSP context. This model is developed by translating the Economic Production Quantity (EPQ) model with two key modules [7] to ELSP context. The mathematical model to solve this particular problem is discussed in Section 2. The mathematical model includes the formula to calculate the total cost $C_i$ and the individual cycle time $T_i$ for each item under Common Cycle (CC) approach. In Section 3, the numerical example to this problem is developed by modifying Bomberger stamping problem [3] to fit the ELSP context with two imperfect key modules. Along with this particular model, the other two numerical examples of ELSP in perfect production system and ELSP with one imperfect key module are presented. These three numerical examples are then compared to each other in term of cycle times, imperfect production system parameters and the total cost in Section 4 to show the significance of using specific ELSP model with two imperfect key modules for solving this particular problem. Conclusions and recommendations of this paper are discussed in the last section.

## 2. Mathematical Model

### 2.1. Notations and basic assumptions

The following notations are used in this particular ELSP model:

| | |
|---|---|
| $i$ | item index, $i$=1, 2,…, $N$ |
| $d_i$ | demand rate in units per unit time assumed to be deterministic, $i$=1, 2,…, $N$ |
| $p_i$ | production rate in units per unit time assumed to be constant $i$=1, 2,…, $N$ |
| $\rho_i$ | $\rho_i = d_i / p_i$ ,$i$=1, 2,…, $N$ |
| $\kappa$ | $\kappa = 1 - \sum_{i=1}^{N} \rho_i$ $i$=1, 2,…, $N$ |
| $\tau_i$ | $\tau_i = \rho_i T_i$ , processing time per lot, $i$=1, 2,…, $N$ |
| $\sigma_i$ | $\sigma_i = s_i + \tau_i$ , total production time per lot, $i$=1, 2,…, $N$ |
| $s_i$ | setup time per unit of time per production lot, independent of sequence, $i$=1, 2,…, $N$ |
| $A_i$ | setup cost per production lot, $i$=1, 2,…,$N$ |
| $h_i$ | holding cost per unit per unit time, $i$=1, 2,…, $N$ |
| $T_i$ | cycle time for item $i$, $i$=1, 2,…,$N$ |
| $t_i$ | length of the production run for item i, $i$=1, 2,…, $N$ |
| $\alpha_i$ | the percentage of defective items produced if the first KM has shifted to out-of-control state, $i$=1, 2,…, $N$ |
| $\beta_i$ | the percentage of defective items produced if the second KM has shifted to out-of-control state, $i$=1, 2,…, $N$ |
| $X$ | time-to-shift of the first KM, an exponentially distributed random variable with mean $1/\mu$ |
| $Y$ | time-to-shift of the second KM, an exponentially distributed random variable with mean $1/\lambda$ |
| $L$ | Lagrange multiplier, a non-negative number |

In this ELSP model with two imperfect key modules, these assumptions apply:

a. only one item can be processed by the facility
b. setup cost and setup time are required for producing each item, and they are known and independent of the production sequence
c. holding cost is known and constant
d. unit defective cost is known and constant
e. demand rate is constant and known over an infinite planning horizon
f. backorder is not allowed which means all demand must be satisfied, and
g. production facility may deteriorate and shift from ‘in-control’ stage to ‘out-of-control’ stage.

### 2.2. EPQ Model with Two Imperfect Key Modules

Fig. 1 shows four production uptime (τ) segmentations mentioned by [7] in which the shocks may occur. Those are $\Omega_1$, $\Omega_2$, $\Omega_3$ and $\Omega_4$. The *x* axis represents the production time run for the first key module, while the *y* axis represents the production time run for the second key module.

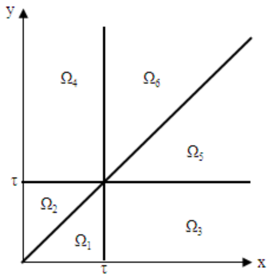


Fig. 1. Production uptime segmentations.

In order to calculate the cost incurred by the non-confirming items produced during the out-of-control states, the expected number of nonconforming items as a function of production uptime ($\tau$) should be formulated. Let $N(\tau)$ be number of non-conforming items, then the $N(\tau)$ for each production uptime segment can be calculated as:

$$N(\tau) = \begin{cases} \alpha(\tau - x) + \beta(\tau - y), & \text{if } (x, y) \in \Omega_1 = \{0 \le y \le x \le \tau\} \\ \alpha(\tau - x) + \beta(\tau - y), & \text{if } (x, y) \in \Omega_2 = \{0 \le x \le y \le \tau\} \\ \beta(\tau - y), & \text{if } (x, y) \in \Omega_3 = \{0 \le y \le \tau \le x\} \\ \alpha(\tau - x), & \text{if } (x, y) \in \Omega_4 = \{0 \le x \le \tau \le y\} \end{cases} \tag{1}$$

Let *X* and *Y* be two random variables exponentially distributed where *X* is the time-to-shift of the first KM and *Y* is the time-to-shift of the second KM. The marginal probability density functions for these two variables are formulated as:
$f_x(x) = \mu e^{-\mu x}$ and $f_y(y) = \lambda e^{-\lambda y}$
Therefore, the expected number of non-conforming items based on the production uptime segmentation and marginal probability density functions is calculated as:

$$E[N(\tau)] = p\iint_{0y}^{\tau\tau} \alpha(\tau - x) + \beta(\tau - y) f_x(x) f_y(y) dxdy + p\iint_{0x}^{\tau\tau} \alpha(\tau - x) + \beta(\tau - y) f_x(x) f_y(y) dxdy + p\iint_{0\tau}^{\tau\infty} \beta(\tau - y) f_x(x) f_y(y) dxdy + p\iint_{\tau 0}^{\infty\tau} \alpha(\tau - x) f_x(x) f_y(y) dxdy \tag{2}$$

After some integration, (2) can be simplified as:

$$E[N(\tau)] = p[(\alpha + \beta)\tau - \frac{\alpha}{\mu}(1 - e^{-\mu\tau}) - \frac{\beta}{\lambda}(1 - e^{-\lambda\tau})] \tag{3}$$

This form (3) is then translated into ELSP context as the base to develop the equation to calculate the quality-related cost.

### 2.3. ELSP model with two imperfect key modules

In order to adjust the EPQ model with two imperfect key modules to ELSP context, the objective, changing variables and constraints to this problem have to be defined. The objective is to minimize the expected total cost $C_i$ for one year for $n$ items that will be discussed in the following section by changing the cycle times $T_i$. The formula to calculate the cycle time can be obtained by deriving the objective function subject to the constraints of ELSP and Kuhn-Tucker necessary condition. Following are the steps undertaken to formulate the cycle time $T$:
Objective function: Minimize the expected total cost

$$\sum_{i=1}^{N} C_i = \sum_{i=1}^{N} f(T) = \sum_{i=1}^{N} \left[ \frac{A_i}{T} + H_i T + Q_i \right] \tag{4}$$

where:

$$Q_i = u_i[(\alpha_i + \beta_i) d_i T - \frac{\alpha_i p_i}{\mu_i}(1 - e^{-\mu_i \rho_i T}) - \frac{\beta_i p_i}{\lambda_i}(1 - e^{-\lambda_i \rho_i T})] \tag{5}$$

by changing the decision variable of cycle time $T$ subject to the constraints:

$(\sum_{i=1}^{N} \frac{s_i}{T}) - \kappa \leq 0$ or in form of $g$ function $g = (\sum_{i=1}^{N} \frac{s_i}{T}) - \kappa$

$T \geq 0$

$L \geq 0$ corresponding to Kuhn-Tucker necessary condition

$L.g = 0$

In Kuhn-Tucker necessary condition, following equation applies:

$$\frac{\delta f}{\delta T} + L.\frac{\delta g}{\delta T} = 0 \tag{6}$$

Following the necessary condition in (6) we obtain:

$$\frac{\delta f(T)}{\delta T} = -\frac{A_i}{T^2} + H_i + u_i(\alpha_i + \beta_i) d_i - u_i \frac{\alpha_i . p_i}{\mu_i} \mu_i \rho_i e^{-\mu_i \rho_i T} - u_i \frac{\beta_i . p_i}{\lambda_i} \lambda_i \rho_i e^{-\lambda_i \rho_i T} \tag{7}$$

$$\text{and } \frac{\delta g}{\delta T} = -\frac{s_i}{T^2} \tag{8}$$

Combining (7) and (8) under the Kuhn-Tucker necessary condition, we obtain:

$$-\frac{A_i}{T} + H_i + u_i(\alpha_i + \beta_i) d_i - u_i \frac{\alpha_i . p_i}{\mu_i} \mu_i \rho_i e^{-\mu_i \rho_i T} - u_i \frac{\beta_i . p_i}{\lambda_i} \lambda_i \rho_i e^{-\lambda_i \rho_i T} - L \frac{s_i}{T^2} = 0 \tag{9}$$

$T$ is solved as:

$$T = \sqrt{\frac{A_i + L.s_i}{H_i + u_i[(\alpha_i + \beta_i) d_i - \alpha_i d_i e^{-\mu_i \rho_i T} - \beta_i d_i e^{-\lambda_i \rho_i T}]}} \tag{10}$$

Even though the closed form of $T$ cannot be obtained, the formula (10) can still be used in finding initial value of $T$ to enhance the searching of optimum cycle time that minimizes the total cost. Algorithm 1 and Algorithm 1-1 are used to calculate the optimum cycle time and minimum total cost for one year, respectively.

**Algorithm 1**

*Step 1.* Set $k$=1 and $\varepsilon$=$10^{-6}$ (or any prescribed small quantity); $k$ and $\varepsilon$ indicate iteration number and prescribed small quantity, respectively.
*Step 2.* Set initial Lagrange multiplier $L$as 0

*Step 3*. Set the initial cycle time $T_k$ as 0
*Step 4*. Calculate the new cycle time $T_{new}$ by using (10)
*Step 5*.If $T_k$=$T_{new}$, go to *Step 8*.Otherwise, go to *Step 6*.
*Step 6*.Set $k$=$k$+1
*Step 7*.Set $T_k$=$T_{new}$, go to *Step 4*
*Step 8*.If $\sum_{i=1}^{N}(s_i + \frac{d_i T^*}{p_i}) \leq T^*$ , go to *Step 10*. Otherwise, go to *Step 9*.
*Step 9*.Increase the $L$ to any non-negative number, go to *Step 3*
*Step 10*.If $\left|\sum_{i=1}^{N}(s_i + \frac{d_i T^*}{p_i}) \leq T^*\right| < \varepsilon$ , go to *Step 12*. Otherwise, go to *Step 11*.
*Step 11*.Decrease the $L$ to any non-negative number, then go to *Step 3*
*Step 12*.Calculate Estimated Total Cost for iteration $k$ ($ETC_k$). The calculation of $ETC_k$ is discussed in Algorithm 1-1.
*Step 13*.A single cycle time $T^*$is optimum. Stop.

**Algorithm 1-1**
*Step 1*. Calculate production uptime $\tau_{ik}$, $H_i$ and average setup cost per unit time $A$ as explained in [9] for CC approach.
*Step 2*.Calculate the expected value of $N$ as $E[N(\tau_{ik})] = p_i[(\alpha_i + \beta_i)\tau_{ik} - \frac{\alpha_i}{\mu_i}(1-e^{-\mu_i \tau_{ik}}) - \frac{\beta_i}{\lambda_i}(1-e^{-\lambda_i \tau_{ik}})]$ .
*Step 3*.Calculate the quality-related cost of producing non-conforming items as: $Q_{ik} = u_i . E[N(\tau_{ik})]$.
*Step 4*. Calculate the total cost $C_{ik}$ per day as $C_{ik} = \frac{A_i}{T^*} + (H_{ik} . T^*) + Q_{ik}$ .
*Step 5*. Calculate the total cost $C_{ik}$ for one year as daily $C_{ij}$ calculated in *Step 4* multiplied with 240 days.

## 3. Numerical example

In order to explain the use of the model introduced in the previous section, a numerical example is generated by modifying the Bomberger's stamping problem [3] and adjusting it to ELSP with two imperfect key modules context. Bomberger's stamping problem is taken from metal stamping facility producing a number of different stampings on the same press line. Production shift is based on one-day shift, which counts 8-hours working. There are actually three types of demand with the value of $a_j$ equals to 1, 3 and 4 in $d_{ij}=a_j.d_{0i}$ when $d_{0i}$ equals to 100. As the previous researchers have been using, this research uses $a_j$=4 such that the demand rate per day for the ten items are shown in Table 1.By using the formula of $T$ in (10), the calculation to obtain optimum cycle times for ten item of modified stamping problem [3]as shown in Table 1 is done under Common Cycle (CC) approach.

The objective of common cycle approach is to find a single cycle time $T^*$ applies for all items in order to minimize the total cost while satisfying the demand. The resulted total cost works as the upper bound (UB) to the solution. The objective function in this problem is stated as minimizing the expected total cost in (4) by changing the decision variable of Lagrange multiplier $L$ subject to the constraints. Following through Algorithm 1 and Algorithm 1-1, the resulted expected total cost for all items in one year is calculated as \$247,592.4. The optimum cycle time for all ten items is 31.892 days while the optimum Lagrange multiplier $L$ that minimizes the total cost is 31149146.5.

Table 1. Modified bomberger stamping problem for elsp with two imperfect key modules.

| Item | Demand Rate (units/day) | Production Rate (units/day) | % Defected Items of KM1 | % Defected Items of KM2 | $\mu$ | $\lambda$ | Setup Time (day) | Setup Cost | Piece Cost |
|---|---|---|---|---|---|---|---|---|---|
| $i$ | $d$ | $p$ | $\alpha$ | $\beta$ | | | $s$ | $A$ | $c$ |
| 1 | 400 | 30,000 | 0.025 | 0.015 | 0.0167 | 0.0185 | 0.125 | 15 | 0.0065 |
| 2 | 400 | 8,000 | 0.015 | 0.010 | 0.0179 | 0.0179 | 0.125 | 20 | 0.1775 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 800 | 9,500 | 0.013 | 0.013 | 0.0167 | 0.0172 | 0.25 | 30 | 0.1275 |
| 4 | 1600 | 7,500 | 0.010 | 0.010 | 0.0172 | 0.0167 | 0.125 | 10 | 0.1 |
| 5 | 80 | 2,000 | 0.015 | 0.025 | 0.0167 | 0.0179 | 0.5 | 110 | 2.785 |
| 6 | 80 | 6,000 | 0.013 | 0.015 | 0.0185 | 0.0167 | 0.25 | 50 | 0.2675 |
| 7 | 24 | 2,400 | 0.013 | 0.013 | 0.0179 | 0.0179 | 1 | 310 | 1.5 |
| 8 | 340 | 1,300 | 0.010 | 0.015 | 0.0172 | 0.0167 | 0.5 | 130 | 5.9 |
| 9 | 340 | 2,000 | 0.013 | 0.025 | 0.0167 | 0.0172 | 0.75 | 200 | 0.9 |
| 10 | 400 | 15,000 | 0.025 | 0.013 | 0.0179 | 0.0167 | 0.125 | 5 | 0.04 |

## 4. Result analysis

As it can be seen in Table 2, the more imperfect production system parameters involved in ELSP case, the higher the total cost in one year is. Under the same approach, the total cost escalates from ELSP in perfect production system to imperfect production system with one key module, and from one key module to two key modules. Thus, the ELSP model in Imperfect Production System with Two Key Modules is verified since the involvement of more imperfect production system parameters can be reflected on the total cost.

Table 2. Total costs and cycle times of ELSP cases.

| ESLP Case | Approach | Cycle Time (T) | Total Cost in One Year($/year) | Imperfect Production System Parameters |
|---|---|---|---|---|
| Perfect Production System | Common Cycle | $T^{*}$=42.75 | 9,879 | None |
| Imperfect Production System with One Key Module | Common Cycle | $T^{*}$=32 | 23,770 | $\alpha$, $\theta$, $u$ |
| Imperfect Production System with Two Key Modules | Common Cycle | $T^{*}$=31.892 | 247,592.43 | $\alpha$, $\beta$, $\mu$, $\lambda$, $u$ |

The second verification is about setting the value of imperfect production system parameters in ELSP in imperfect production system with two key modules into zero to check if it is consistent with ELSP in perfect production system. There are five imperfect production system parameters in ELSP with two imperfect key modules which are $\alpha$, $\beta$, $\mu$, $\lambda$ and $u$. By using the model of ELSP in imperfect production system with two key modules and setting the value of these parameters into zero, the expected number of non-conforming items $E[N(\tau)]$ is equal to zero. When the value of expected number of non-conforming items turns to zero, so does the quality related cost $Q_i$. Therefore, the cost structure of this model becomes:

$$C_i = f(T_i) = \frac{A_i}{T_i} + H_i.T_i \quad (11)$$

where:

$$H_i = \frac{h_i.d_i.(1-\rho_i)}{2} \quad (12)$$

It is proven that when the imperfect production system parameters of ELSP in imperfect production system with two key modules are turned into zero, the total cost will be transformed to ELSP in perfect production system. This shows that the ELSP model in imperfect production system with two key modules is verified.

As in Table 3, under the Common Cycle approach, the optimum cycle time generated from ELSP in perfect production system model is 42.75 days for all items. When this cycle time is implemented in ELSP context with two imperfect key modules, the resulted total cost for one year is $431,536.4 or 74.3% higher than the total cost for one year generated from optimum cycle times in ELSP model with two imperfect modules which is $247,592.4. This shows the significance of this proposed model in term of total cost.

Table 3. Total cost incurred when applying cycle times of ELSP with perfect production system in ELSP with two imperfect key modules.

| ELSP Case | Approach | Total Costs ($) | |
|---|---|---|---|
| | | *Applying Cycle Times under Perfect Production System* | *Applying Cycle Times under Imperfect Production System with 2 KMs* |
| Imperfect Production System with Two Key Modules | Common Cycle | 431,536.4 | 247,592.4 |

As it is shown in Table 3, the result under CC approach shows that when facing an imperfect production system with two key modules under ELSP context, it is necessary to use the model of ELSP with two imperfect key modules to solve the optimum cycle times instead of ELSP with perfect production system to avoid making decision errors that may lead to high total cost.

## 5. Concluding remarks

This paper translates the Economic Production Quantity (EPQ) model with two imperfect key modules introduced in [7] to the Economic Lot Scheduling Problem (ELSP) context by formulating the formula and algorithms to calculate the cycle time and the total cost applying the Common Cycle (CC) approach. Modifying the stamping problem introduced in [3] into ELSP context with two imperfect key modules, the numerical example is given to show how the model is used. A single cycle time of 31.892 days is calculated by using the model. In order to show the significance of using the model, the cycle time of ELSP in perfect production system is applied for ELSP with two imperfect key modules. The resulted total cost for one year is $431,536.4 or 74.3% higher than the total cost for one year generated from optimum cycle times in ELSP model with two imperfect modules which is $247,592.4. Under this specific theme on ELSP, further research may be done by developing the model under Basic Period (BP) and Time-Varying Lot Size approach to achieve lower total cost in one year.

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