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by The Jin Ai

A Joint Replenishment Inventory Model with Lost Sales

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Abstract. This paper deals with two items joint replenishment inventory problem, in which the demand of each items are constant and deterministic. Inventory replenishment of items is conducted periodically every T time intervals. Among of these replenishments, joint replenishment of both items is possible. It is defined that item i is replenished every $Z_i T$ time intervals. Replenishment of items are instantaneous. All of shortages are considered as lost sales. The maximum allowance for lost sales of item i is S_i. Mathematical model is formulated in order to determining the basic time cycle T, replenishment multiplier Z_i, and maximum lost sales S_i in order to minimize the total cost per unit time. A solution methodology is proposed for solve the model and a numerical example is provided for demonstrating the effectiveness of the proposed methodology.

1. Introduction

In the research on multi-item inventory policy decision problems, there are many possible ways to be conducted in order to obtain cost efficiency. One of the possible way is conducting joint replenishment of items. Some researchers in the past have been proposed a problem called the joint replenishment problem (JRP) in order to address multi item inventory system, in which several items may be jointly replenished from the same supplier [1][2][3][4].

After the basic joint replenishment problem proposed [5], some researchers have been working on the methodology to solve this problem [6][7][8]. While, other researchers have been working on the variants of this problem, such as stochastic demand [9][10], quantity discount [11], discrete time replenishment [12], and continuous unit cost decrease [13].

The condition of shortage, i.e. whenever demand is exist while the inventory level is zero, is common in the inventory system. Especially in the discussion on stochastic demand, shortage is common to be included in the model. However, to the best of authors' knowledge in the JRP model, all shortage is usually treated as backorder. It is well known in the practice that some customers are not willing to wait whenever no stock in the inventory. In other words, the shortage is not backorder but lost sales. Therefore, it is necessary to develop JRP model that treated shortage as lost sales.

The remainder of this paper is organized as follows: Problem and mathematical formulation is defined in section 2. After that, solution methodology of the proposed mathematical model is proposed in section 3. Section 4 presents some numerical examples. Finally, some the conclusion of this study is presented with some suggestions for further research in this research area.

2. Problem and Mathematical Formulation

The problem can be formulated as follow. An n items inventory problem is considered here, where the demand of each items are constant and deterministic. The demand rate of item i is D_i units per unit time. Replenishment of item i is instantaneous with size of Q_i units. A joint replenishment is conducted periodically every T time intervals. However, all items may not be included in each replenishment. Item i is only included every Z_iT time intervals. In other word, Z_iT is the cycle time of item i. A replenishment incurs of major ordering cost $\$K$ and minor ordering cost $\$k_i$ for every item i included in the replenishment. The inventory holding cost of item i is $\$h_i$ per unit per unit time. All of shortages are considered as lost sales. The maximum allowance for lost sales of item i is S_i and the lost sales cost of item i is $\$p_i$ per unit. The decision to be taken in this situation is determining the basic time cycle T, replenishment multiplier Z_i, and lost sales quantity S_i in order to minimize total cost per unit time.

Inventory level of this problem situation with 2 items, $Z_1=1$, and $Z_2=2$ is illustrated in Figure 1. The total lost sales of item i is S_i can be calculated as $t_i D_i$. Therefore, the optimization problem can be converted into minimizing total cost C as the function of T, Z_i, and t_i.
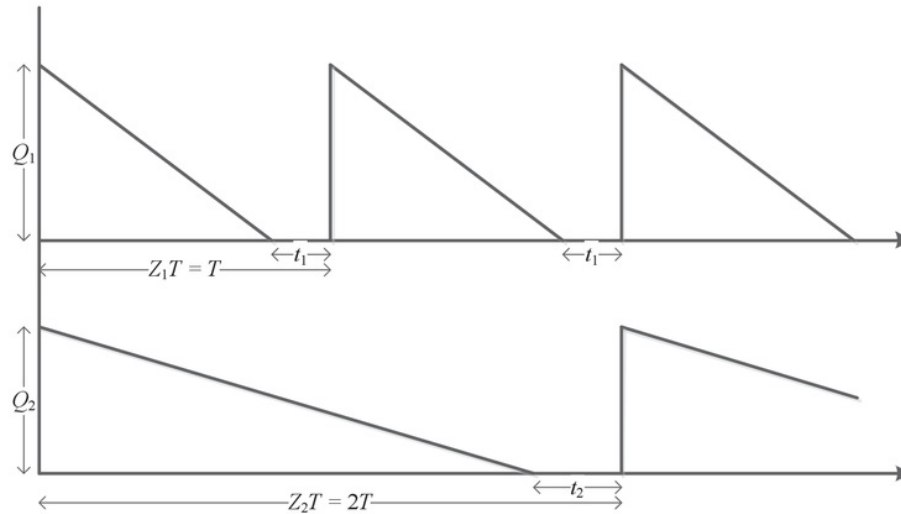

Figure 1. Illustration of inventory level with 2 items, $Z_1=1$, $Z_2=2$.

The total setup cost per unit time can be formulated as follow

$$C_1 = \frac{1}{T}\left(K + \sum_i \frac{k_i}{Z_i}\right) \tag{1}$$

The total holding cost per unit time can be formulated as follow

$$C_2 = \sum_i \frac{D_i (Z_iT - t_i)^2 h_i}{2Z_iT} \tag{2}$$

The total lost sales cost per unit time can be formulated as follow

$$C_3 = \sum_i \frac{D_i t_i p_i}{Z_iT} \tag{3}$$

Therefore, the total cost can be summarized as follow

$$C = C_1 + C_2 + C_3$$

$$= \frac{1}{T}\left(K + \sum_i \frac{k_i}{Z_i}\right) + \sum_i \frac{D_i(Z_iT - t_i)^2 h_i}{2Z_iT} + \sum_i \frac{D_i t_i p_i}{Z_iT} \tag{4}$$

After some algebra, Equation (4) can be written as

$$C = \frac{1}{T}\left(K + \sum_i \frac{k_i + D_i p_i t_i}{Z_i}\right) + \sum_i \left(\tfrac{1}{2}D_i h_i Z_i T - D_i h_i t_i + \frac{D_i h_i t_i^2}{2Z_iT}\right) \tag{5}$$

It is also seen from Figure (1) that following constraints are necessary:

$$T \geq 0 \tag{6}$$

$$0 \leq t_i \leq TZ_i, \quad \forall i \tag{7}$$

$$Z_i \in \text{integer}, \quad \forall i \tag{8}$$

3. Solution Procedure

It is noted that the objective function in Equation (5) consists of two types of variables, which are real variables, i.e. T and t_i, and integer variables, i.e. Z_i. Therefore, multi variables classical optimization technique cannot be directly applied here. However, some of the classical optimization technique can be partially applied.

Let first consider the situation of fixed value of t_i and Z_i. In this situation, the objective function can be considered as function of single variable T. Therefore, necessary condition to obtain minimum value of C is the first derivative of C with respect to T is equal to zero.

$$\frac{d}{dT}C = -\frac{1}{T^2}\left(K + \sum_i \frac{k_i + D_i p_i t_i}{Z_i}\right) + \sum_i \left(\tfrac{1}{2}D_i h_i Z_i - \frac{D_i h_i t_i^2}{2Z_iT^2}\right) = 0 \tag{9}$$

After some algebra, the expression of optimal T can be obtained as follow

$$(T^*)^2 = \frac{2\left(K + \sum_i \frac{k_i}{Z_i} + \sum_i \frac{D_i h_i t_i^2}{2Z_i} + \sum_i \frac{D_i p_i t_i}{Z_i}\right)}{\sum_i (D_i h_i Z_i)} \tag{10}$$

Therefore,

$$T^* = \sqrt{\frac{2\left(K + \sum_i \frac{k_i}{Z_i} + \sum_i \frac{D_i h_i t_i^2}{2Z_i} + \sum_i \frac{D_i p_i t_i}{Z_i}\right)}{\sum_i (D_i h_i Z_i)}} \tag{11}$$

Second, let consider the situation of fixed value of T and Z_i. In this situation, the objective function can be considered as function of multi variable t_i. Therefore, necessary condition to obtain minimum value of C is the first derivative of C with respect to t_i is equal to zero.

$$\frac{\partial}{\partial t_i}C = \frac{1}{T}\left(\frac{D_i p_i}{Z_i}\right) - D_i h_i + \frac{D_i h_i t_i}{Z_iT} = 0 \tag{12}$$

After some algebra, the expression of optimal T can be obtained as follow

$$t_i^* = Z_i T - \frac{p_i}{h_i} \tag{13}$$

It is noted that due to constraint (7), which is $t_i \geq 0$, following expression is also required

$$T \geq \frac{p_i}{Z_i h_i} \tag{14}$$

Next, let consider the situation of fixed value of T and t_i. In this situation, the objective function can be considered as function of multi variable Z_i, in which all variable Z_i are integer. To find an optimal value of Z_j, let consider the situation whenever the other values of Z_i ($i \neq j$) are fixed. Since Z_j is integer, following condition is required to minimize C.

$$C\left(Z_1, Z_2, ..., Z_j - 1, ..., Z_n\right) \leq C\left(Z_1, Z_2, ..., Z_j, ..., Z_n\right) \leq C\left(Z_1, Z_2, ..., Z_j + 1, ..., Z_n\right) \tag{15}$$

Substituting equation (5) into condition above and after some algebra, following condition is obtained for optimal value of Z_i

$$Z_i^*\left(Z_i^* - 1\right) \leq \frac{2}{T^2}\left(\frac{t_i^2}{2} + \frac{1}{h_i}\left(\frac{k_i}{D_i} + t_i p_i\right)\right) \leq Z_i^*\left(Z_i^* + 1\right) \tag{16}$$

Based on some optimality conditions describe above, following iterative procedure is proposed in order to find the optimal values of T, t_i, and Z_i.

Step 1.   Set all $t_i = 0$ and all $Z_i = 1$. Set iteration index $j = 1$.
Step 2.   Calculate T using equation 11, with considering constraints in equation 14. In particular, for any item i, if current value of T is smaller than $p_i / Z_i h_i$ then update $T = p_i / Z_i h_i$
Step 3.   Update the values of t_i using equation 13
Step 4.   Find the optimal values of Z_i using conditions in equation 16
Step 5.   If updated values of T, t_i, and Z_i are similar with previous iteration, then stop. Otherwise, return to Step 2.

4. Numerical Example
A numerical example is considered here in order to show the application of the proposed solution procedure. Let consider the joint replenishment problem with 2 items. Demand of item 1, D_1, is 900 unit/year and demand of item 2, D_2, is 600 unit/year. Holding cost of item 1 and 2 are \$4/unit/year and \$2.5/unit/year, respectively. Major setup cost is \$100/order, while minor setup cost of item 1 and 2 are \$10/order and \$7/order, respectively. The lost sales cost of item 1 and 2 are \$2/unit and \$1/unit, respectively.

Following proposed procedure described in previous section, the iteration steps of this example are presented in the Table 1. It is obtained that the optimal values of decision variables are $T = 0.5$, $t_1 = 0$, $t_2 = 0.1$, $Z_1 = 1$, $Z_1 = 1$ with total cost $C = \$1494$/year.

5. Conclusion
This paper is successfully presented a mathematical model for joint replenishment inventory system with lost sales. The methodology for solving the model is also proposed. For the future work, the extension of this model and solution methodology is expected in the area of stochastic demand. Other possibility is to extend this model to partial backlog case, that is also common in the inventory practice.

Table 1. Iteration Steps to Find T, t_1, t_2, Z_1, and Z_2.

Iteration	T	t_1	t_2	Z_1	Z_2	C
1	1.4096	1.0000	1.0000	1	1	2089.12
2	1.3451	0.9096	1.0096	1	1	2071.12
3	1.2774	0.8451	0.9451	1	1	2054.53
4	1.2059	0.7774	0.8774	1	1	2035.16
5	1.1298	0.7058	0.8058	1	1	2012.10
6	1.0482	0.6298	0.7298	1	1	1984.03
7	0.9598	0.5482	0.6482	1	1	1948.81
8	0.8623	0.4597	0.5597	1	1	1902.76
9	0.7523	0.3622	0.4622	1	1	1838.85
10	0.6231	0.2522	0.3522	1	1	1741.25
11	0.5000	0.1230	0.2230	1	1	1571.26
12	0.5000	0.0000	0.1000	1	1	1494.00
13	0.5000	0.0000	0.1000	1	1	1494.00

References

[1]   Khouja M and Goyal S 2008 *European Journal of Operational Research* **186** 1–16
[2]   Moon IK, Goyal SK and Cha BC 2008 *Int. Journal of Systems Science* **39** 629–37
[3]   Tsai CY, Tsai CY and Huang PW 2009 *Int. Journal of Production Economics* **117** 30–41
[4]   Wang L, He J, Wu D and Zeng YR 2012 *International Journal of Production Economics* **135** 190–8
[5]   Shu FT 1971 *Management Science* **17** B-406–10
[6]   Kaspi M and Rosenblatt MJ 1991 *Int. Journal of Production Research* **29** 107–14
[7]   Porras E and Dekker R 2006 *European Journal of Operational Research* **174** 1595–615
[8]   Olsen AL 2005 *Computers and Industrial Engineering* **48** 223–35
[9]   Johansen SG and Melchiors P 2003 *Journal of the Operational Research Society* **54** 283–90
[10]   Atkins DR and Iyogun O 1988 Management Science **34** 791–6
[11]   Cha BC and Moon IK 2005 *OR Spectrum* **27** 569–81
[12]   Klein CM and Ventura JA 1995 *Journal of the Operational Research Society* **46** 643–57
[13]   Khouja M, Park S and Saydam C 2005 *Int. Journal of Production Research* **43** 311–26

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