Paper 38

by The Jin Ai

An Inventory Decision Model of Two Products with Vector Autoregressive Demand

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Abstract. An inventory system that consist of two different products is considered in this paper. Demand of these products have purchase dependent power, i.e. demand of product A is affecting demand of product B, and vice versa. Then, a vector autoregressive VAR(1) model is being used as the reference forecasting model for the demand pattern. This forecasting model is being used as a basis for determining inventory policy for two products with purchase dependent power. An inventory decision model regarding when and how much to order of these products is developed in order to minimize total inventory cost consists of ordering and holding cost. Possibility to reducing order cost by joining replenishment of these products also being evaluated.

Keywords: purchase dependent power, vector autoregressive, inventory decision model, joint replenishment

1. INTRODUCTION

The purchase dependency is appeared in the retail industry. According to Bala (2008a), there are two types of purchase dependencies, within transaction and inter-transactional. Purchase dependency within transaction occurs in a single purchase transaction, while an inter-transactional purchase dependency occurs between a single purchase transaction and a previous purchase transaction. It is also studied that consumer behavior that leads to purchase dependency on products can be used as a decision-making tool and the preparation of marketing strategy, product, product arrangement on the display rack, and inventory arrangement of retail company's goods (Bala, 2008b). One possible way to determine the purchase dependency is using regression analysis (Bala, 2010).

Consideration of purchase dependence into inventory decision making has been studied by Park and Seo (2013). They developed the model purchase dependence with reference continuous review (Q, R) model and periodic review (R, T) model. They able to prove the importance of purchase dependence in determining inventory policy by showing the effect of purchase dependence on the total inventory cost, i.e. by considering purchase dependence while determining inventory policy can decrease inventory cost.

In different way, Ai, et al. (2016) developed forecasting model of purchase dependence demand by using the vector autoregressive VAR(1) model. They confirmed the model from a clothing retail company and conducted several experiments to study the nature of the developed VAR(1) model. It is shown that the VAR(1) equation in the purchase dependence product can be used for products with linear increasing and linear decreasing linear demand.

In this paper, we tried to develop a methodology for determining the inventory policy of two products when the demands are following the VAR(1) forecasting model.

2. METHODOLOGY DEVELOPMENT

Following Ai, et al. (2016), the vector autoregressive VAR(1) model can be mathematically formulated as follow

$$Y_{1,t} = \delta_1 + \phi_{1,1} Y_{1,t-1} + \phi_{1,2} Y_{2,t-1} + \varepsilon_{1,t} \quad (1)$$

$$Y_{2,t} = \delta_2 + \phi_{2,1} Y_{1,t-1} + \phi_{2,2} Y_{2,t-1} + \varepsilon_{2,t} \quad (2)$$

where

$Y_{i,t}$: demand of product i at period t

$\phi_{i,j}$: an autoregressive coefficient, which represents the effect of demand product i at period $(t-1)$ on demand product j at period t

δ_i : an autoregressive coefficient, which represents the part of demand product i at period t that is not affected by past demand

$\varepsilon_{i,t}$: error term of autoregressive equation i, which represents the part of demand product i at period t that cannot explained by the autoregressive equation

From equations (1) and (2), it is known that the demand of product 1 and 2 over certain periods of time T can be forecasted with the knowledge of parameters $\phi_{i,j}$, δ_i, and initial value of demand $Y_{i,0}$. If the error terms of these forecast model is dropped, the demands can be considered as deterministic and dynamic. Therefore, in order to determine an inventory policy for the two products following equations (1) and (2), dynamic economic lot size model of Wagner and Whitin (1958) is selected as reference model.

Two alternatives of procedure are proposed here. The first alternative is independently applying the Wagner-Whitin model for each product for obtaining the inventory policy, i.e. the replenishment schedule for each product. The second alternative is modifying the Wagner-Whitin model so that the replenishment schedule for all products are the same.

For the first alternative, the basic total cost equations for determining the inventory policy is as follow:

$$TC = \min \begin{cases} s_1 + h_2 Y_{i,2} + \cdots + h_t Y_{i,t} \\ a_1^* + s_2 + h_3 Y_{i,3} + \cdots + h_t Y_{i,t} \\ \vdots \\ a_{t-1}^* + s_t \end{cases} \quad (3)$$

where:

h_t : holding cost for period t

s_t : setup cost for period t

a_t^* : optimal cost at period t

While for the second alternative, the basic total cost equations for determining the inventory policy consists of the holding cost for all products, as follow:

$$TC = \min \begin{cases} s_1 + h_2 Y_{1,2} + h_2 Y_{2,2} + \cdots + h_t Y_{1,t} + h_t Y_{2,t} \\ a_1^* + s_2 + h_3 Y_{1,3} + h_3 Y_{2,3} \cdots + h_t Y_{1,t} + h_t Y_{2,t} \\ \vdots \\ a_{t-1}^* + s_t \end{cases} \quad (4)$$

3. NUMERICAL EXAMPLE

Let consider two products VAR(1) forecasting model with following equations:

$$Y_{1,t} = -10.715 + 0.553 Y_{1,t-1} + 1.328 Y_{2,t-1} \quad (5)$$

$$Y_{2,t} = 23.128 + 0.337 Y_{1,t-1} - 0.050 Y_{2,t-1} \quad (6)$$

Based on these equations, the demand of each product can be written as Table 1.

Table 1: Demand of products

Period	Product 1	Product 2
1	177	52
2	156	80
3	182	72
4	185	81
5	199	81
6	208	86
7	219	89
8	228	93
9	238	95
10	248	98
11	257	102
12	266	105

For simplest test case, it is assumed that the unit holding cost for all product and all period is equal to 1. Various setup cost and holding cost ratio are considered, which are 1:100 up to 1:1000. Implementing the alternatives for obtaining the replenishment schedule, it is obtained that the replenishment schedule for product 1 and 2 for the first alternative solution are shown in Table 2 and 3, respectively. While the replenishment schedule for both product 1 and 2 for the second alternative solution is shown in Table 4.

The comparison of total cost between the first and second alternatives are presented in Figure 1. It is noted that total cost of the second alternative (blue lines) is smaller than the total cost of the first alternative (orange lines) across the setup to holding cost ratio.

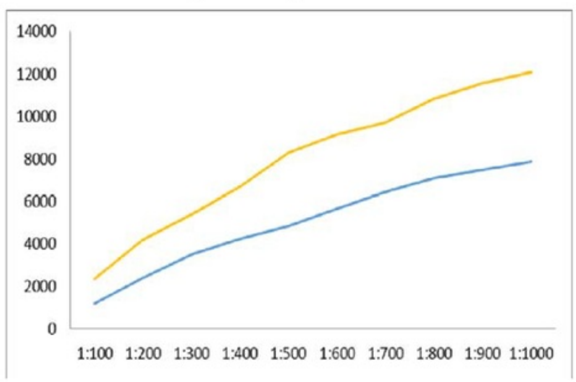

Figure 1: Comparison of Total Cost

Table 2: Replenishment schedule for product 1 for the first alternative

Period / Ratio	1	2	3	4	5	6	7	8	9	10	11	12	No. Replenishment
1:100	x	x	x	x	x	x	x	x	x	x	x	x	12
1:200	x		x		x	x	x	x	x	x	x	x	10
1:300	x		x		x		x		x		x		6
1:400	x			x		x		x		x	x		6
1:500	x			x			x			x		x	5
1:600	x				x			x			x		4
1:700	x				x			x			x		4
1:800	x					x				x			3
1:900	x					x				x			3
1:1000	x					x				x			3

Table 3: Replenishment schedule for product 2 for the first alternative

Period / Ratio	1	2	3	4	5	6	7	8	9	10	11	12	No. Replenishment
1:100	x		x		x		x		x	x	x	x	8
1:200	x			x			x			x		x	5
1:300	x				x			x			x		4
1:400	x					x				x			3
1:500	x						x					x	3
1:600	x						x					x	3
1:700	x							x					2
1:800	x							x					2
1:900	x								x				2
1:1000	x								x				2

Table 4: Replenishment schedule for both products 1 and 2 for the second alternative

Period / Ratio	1	2	3	4	5	6	7	8	9	10	11	12	No. Replenishment
1:100	x	x	x	x	x	x	x	x	x	x	x	x	12
1:200	x	x	x	x	x	x	x	x	x	x	x	x	12
1:300	x		x		x		x	x	x	x	x	x	9
1:400	x		x		x		x		x		x		6
1:500	x		x		x		x		x		x		6
1:600	x			x			x		x		x		5
1:700	x			x			x			x		x	5
1:800	x				x			x			x		4
1:900	x				x			x			x		4
1:1000	x				x			x			x		4

4. CONCLUDING REMARKS

This paper demonstrates that Wagner-Whitin algorithm and its modification can be applied for determining the inventory policy, i.e. the replenishment schedule for two products that has purchase dependence property and being forecasted using VAR(1) model. The numerical examples show that the second alternative, which make the replenishment schedule for all products are the same, provides smaller total cost than the first one.

It is important that error term is being considered in the future research. Furthermore, the study should compare in detail the effect on considering or not the purchase dependence property in the inventory model, i.e. to prove whether the total inventory cost is reduced when purchase dependence is being considered

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