CHAPTER 2
LITERATURE REVIEW AND BASE THEORY

2.1. Literature Review

2.1.1. Frequency Domain Decomposition (FDD)

The frequency-domain decomposition (FDD) technique that is based on singular value decomposition (SVD) of the spectral density (SD) matrix, makes it possible to analyze cases with closely spaced modes. Further, the FDD makes the frequency-domain technique more user friendly because it concentrates all information in one single plot; that is the plot of singular values of the SD matrix.

The technique was introduced by Brincker et al. (2000) as improvement of basic "peak-picking" method to better separation of closely spaced modes and offer method to more accurately estimate the damping ratio. From the simulation proof given in the paper, FDD method can give very accurate result on natural frequency (0.1% difference in 20% noise) and quite good damping ratio (4-5% difference in 20% noise).

In analysis of *Operational Modal Analysis of Large Bridge* made by Schanke (2015) FDD is compared with others method both from time domain and frequency domain. Compared with other basic method as SOBI and peak-picking, FDD give the best result in estimating natural frequency for basic method and only better by more advanced method such as Cov-SSI while quite computationally efficient.

Other analysis investigating damping ratio is of *Damping Estimation of Large Wind-Sensitive Structures* made by Cheynet et al. (2016). In the paper FDD method is compared with Cov-SSI method for measurement of long-span suspension brige by comparing measurement result to a numerical model. From the experiment , AFDD algorithm was observed to estimate the MDRs with a larger bias than the SSI-COV method. This suggests that the frequency-domain based approach is not well suited for the modal parameters identification of long suspension bridges with eigen-frequencies around and below 0.1 Hz.

2.1.2. Ambient Vibration Test

Ambient vibration test or OMA is very attractive because tests are cheap and fast, and they do not interfere with the normal use of the structure. Moreover, the identified modal parameters are representative of the actual behavior of the structure in its operational conditions, since they refer to levels of vibration actually present in the structure and not to artificially generated vibrations.

In some cases, such as testing of historical structures (where it reduces the invasiveness and the risk of damage) or vibration-based health assessment and monitoring (where the replacement of the artificial excitation with ambient vibrations makes it suitable for automation), OMA outperforms EMA.

2.1.3. Accelerometer Sensor Micro-Electro Mechanichal System (MEMS)

Beskhyroun dan Ma (2012), in research of "*Low-Cost Accelerometer for Experimental Modal Analysis*" able to using accelerometer sensor X6-1A USB manufactured by Gulf Coast Data Concept (GCDC) record several aftershocks response of three high risereinforced concrete buildings in Christchurch city, New Zealand after the city with hit by two major earthquakes. The recorded data produced very accurate estimates of the modal parameters of the instrumented buildings. Two commonly used system identification techniques, the frequency domain peak pick method and the more advanced time domain stochastic subspace identification method were implemented to extract modal parameters.

2.2. Basic Theory

2.2.1. Autocorrelation

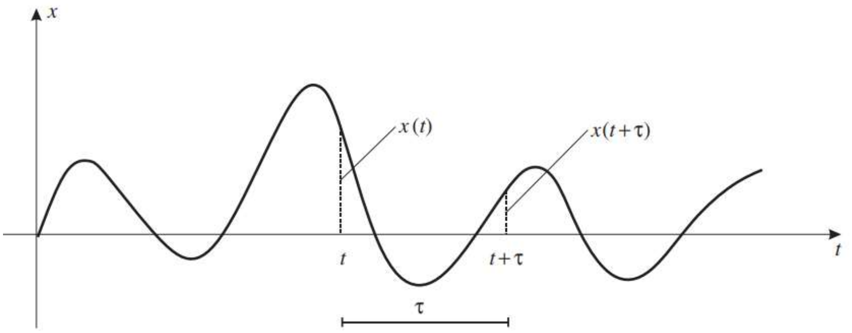

Fig 2.1 Random signal correlation

In varying signal $x(t)$ as shown in Figure 2.1, if the points are close, then the correlation is high, and as the separation between points increases then the correlation is lower – and finally if the points are far apart – then the correlation is for practical purposes zero. C two points $x(t)$ and $x(t+\tau)$ with a time separation τ in between them (see Figure 2.1). By taking the variable x as $x(t)$ and the variable y as $x(t+\tau)$ Eq. (2.1) can be used to define the autocorrelation function defined as

$$R_x(\tau) = \mathrm{E}\left[x(t)x(t+\tau)\right] \tag{2.1}$$

The autocorrelation is in practice obtained by using time averaging version of the function such as

$$R_x(\tau) = \frac{1}{T}\int_0^T x(t)x(t+\tau)dt \tag{2.2}$$

2.2.2. Power Spectral Density

The auto spectral density function for a time series $x(t)$ is defined as the Fourier transform of the correlation function $R_x(\tau)$

$$G_x(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_x(\tau)e^{-i\omega\tau}d\tau \tag{2.3}$$

conversely the correlation function can be found from the inverse relation

$$R_x(\tau) = \int_{-\infty}^{\infty} G_x(\omega)e^{i\omega\tau}d\omega \tag{2.4}$$

2.2.3. Theory Background of Frequency Domain Decomposition

According to Bendat and pearsol, the relationship between the unknown inputs $x(t)$ and the measured responses $y(t)$ can be described as:

$$G_{yy}(j\omega) = \overline{H}(j\omega)G_{xx}(j\omega)H(j\omega)^T \tag{2.5}$$

Where $G_{xx}(j\omega)$ is the $r \times r$ Power Spectral Density (PSD) matrix of the input, r is the number of inputs, $G_{yy}(j\omega)$ is the $m \times m$ PSD matrix of the responses, m is the number of responses, $H(j\omega)$ is the $m \times r$ Frequency Response Function (FRF) matrix, and " – " and superscript T denote complex conjugate and transpose, respectively.

The FRF can be written in partial fraction, i.e. pole/residue form

$$H(j\omega)=\sum_{k=1}^{n}\frac{R_k}{j\omega-\lambda_k}+\frac{\overline{R}_k}{j\omega-\overline{\lambda}_k} \tag{2.6}$$

Where n is the number of modes, λ_k is the pole and R_k is the residue

$$R_k=\phi_k\gamma_k^T \tag{2.7}$$

Where ϕ_k,γ_k is the mode shape vector and the modal participation vector, respectively. Suppose the input is white noise, i.e. its PSD is a constant matrix, i.e. $G_{xx}(j\omega)=C$, then Equation (5) becomes

$$G_{yy}(j\omega)=\sum_{k=1}^{n}\sum_{s=1}^{n}\left[\frac{R_k}{j\omega-\lambda_k}+\frac{\overline{R}_k}{j\omega-\overline{\lambda}_k}\right]C\left[\frac{R_s}{j\omega-\lambda_s}+\frac{\overline{R}_s}{j\omega-\overline{\lambda}_s}\right]^H \tag{2.8}$$

where superscript H denotes complex conjugate and transpose. Multiplying the two partial fraction factors and making use of the Heaviside partial fraction theorem, after some mathematical manipulations, the output PSD can be reduced to a pole/residue form as follows

$$G_{yy}(j\omega)=\sum_{k=1}^{n}\frac{A_k}{j\omega-\lambda_k}+\frac{\overline{A}_k}{j\omega-\overline{\lambda}_k}+\frac{B_k}{-j\omega-\lambda_k}+\frac{\overline{B}_k}{-j\omega-\overline{\lambda}_k} \tag{2.9}$$

where A_k is the k th residue matrix of the output PSD. As the output PSD itself the residue matrix is an $m\times m$ hermitian matrix and is given by

$$A_k=R_kC\left(\sum_{s=1}^{n}\frac{\overline{R}_s^T}{-\lambda_k-\overline{\lambda}_s}+\frac{R_s^T}{-\lambda_k-\lambda_s}\right) \tag{2.10}$$

The contribution to the residue from the k th mode is given by

$$A_k=\frac{R_kC\overline{R}_s^T}{2\alpha_k} \tag{2.11}$$

Where α_k is minus the real part of the pole $\lambda_k = -\alpha_k + j\omega_k$. As it appears, this term becomes dominating when the damping is light, and thus, is case of light damping, the residue becomes propotional to the mode shape vector

$$\begin{aligned} A_k \propto R_k C \overline{R}_k &= \phi_k \gamma_k^T C \gamma_k \phi_k^T \\ &= d_k \, \phi_k \phi_k^T \end{aligned} \tag{2.12}$$

where d_k is a scalar constant. At a certain frequency ω only a limited number of modes will contribute significantly, typically one or two modes. Let this set of modes be denoted by $Sub(\omega)$ .Thus, in the case of a lightly damped structure, the response spectral density can always be written

$$G_{yy}(j\omega) = \sum_{k \in Sub(\omega)}^{n} \frac{d_k \phi_k \phi_k^T}{j\omega - \lambda_k} + \frac{\overline{d}_k \overline{\phi}_k \overline{\phi}_k^T}{j\omega - \overline{\lambda}_k} \tag{2.13}$$

This is a modal decomposition of the spectral matrix. The expression is similar to the results one would get directly from Equation (2.5) under the assumption of independent white noise input, i.e. a diagonal spectral input matrix.

2.2.4. Identification Algorithm

In the Frequency Domain Decomposition (FDD) identification, the first step is to estimate the power spectral density matrix. The estimate of the output PSD $\hat{G}_{yy}(j\omega_i)$ known at discrete frequencies $\omega = \omega_i$ is then decomposed by taking the Singular Value Decomposition (SVD) of the matrix

$$\hat{G}_{yy}(j\omega_i) = U_i S_i U_i^H \tag{2.14}$$

where the matrix $U_i = [u_{i1}, u_{i2}, \mathrm{K}, u_{im}]$ is a unitary matrix holding the singular vectors u_{ij}, and S_i is a diagonal matrix holding the scalar singular values s_{ij}. Near a peak corresponding to the k th mode in the spectrum this mode or may be a possible close mode will be dominating. If only the k th mode is dominating there will only be one term in Equation (2.9). Thus, in this case, the first singular vector u_{i1} is an estimate of the mode shape

$$\hat{\phi} = u_{i1} \tag{2.15}$$

and the corresponding singular value is the auto power spectral density function of the corresponding single degree of freedom system, refer to Equation (2.9). This power spectral density function is identified around the peak by comparing the mode shape estimate $\hat{\phi}$ with the singular vectors for the frequency lines around the peak. As long as a singular vector is found that has high MAC value with $\hat{\phi}$ the corresponding singular value belongs to the SDOF density function.

From the piece of the SDOF density function obtained around the peak of the PSD, thenatural frequency and the damping can be obtained. In this paper the piece of the SDOF PSD was taken back to time domain by inverse FFT, and the frequency and the damping wassimply estimated from the crossing times and the logarithmic decrement of the corresponding SDOF auto correlation function.