CHAPTER 2
LITERATURE REVIEW AND THEORETICAL BACKGROUND

This chapter explains the previous researches in economic lot scheduling problem (ELSP) and the theoretical background of this research. This chapter is divided into three sub-discussions, which are the theoretical backgrounds of ELSP and EPQ model with two imperfect modules; and the literature review based on ELSP five key research themes. Based on these five key research themes, this research position among the other papers in ELSP is explained. Later in this chapter, the previous researches on ELSP are compared one to each other to show the research gap.

2.1. Economic Lot Scheduling Problem

Economic Lot Scheduling Problem (ELSP) was originally arisen from the problem of Economic Manufacturing Quantity (EMQ). EMQ deals with producing individual item in a single facility. The complexity of the problem arises when the production facility is required to produce more than one item at the same time which is physically impossible. Therefore, the items should compete for the same facility. The problem now becomes minimizing the cost incurred of the making repetitive schedule for the items. In ELSP, this problem is solved by obtaining optimum cycle times (T) for the items.

According to Elmaghraby (1978), there are two broad categories of approaches to solve ELSP which are the analytical approach and heuristic approach. The analytical approach tends to find the optimum solution of the restricted version of the problem while in contrast, heuristic approach tends to find good (or sometimes very good) solutions for the problem. There is always a cost suffered for deviating the result from the true optimum solution for both approaches. The basic concept of ELSP can be explained as follows:

Let i be the item index; i=1, 2, …, N. In ELSP, these basic data for each item are usually given:

r_i demand rate in units per unit time assumed to be deterministic, i=1, 2,…, N

p_i production rate in units per unit time assumed to be constant i=1, 2,…, N

s_i setup time per unit of time per production lot, independent of sequence, i=1, 2,…, N

A_i setup cost per production lot, i=1, 2,…, N

h_i holding cost per unit per unit time, i=1, 2,…, N

From these basic data, several other system parameters are derived. These parameters are:

ρ_i $\rho i = \frac{r_i}{p_i}$,i=1, 2,…, N

T_i the unknown cycle time for item i, i=1, 2,…, N

τ_i $\tau_i = \rho_i T_i$, processing time per lot, i=1, 2,…, N

σ_i $\sigma_i = s_i + \tau_i$, total production time per lot, i=1, 2,…, N

q_i the lot size $q_i = r_i . T_i$

η_i the frequency of production $\eta_i = 1/T$

The formula of average total cost per unit time when item i is produced is calculated as:

$$C_i = \frac{A_i}{T_i} + \frac{h_i r_i (1-\rho_i) T_i}{2} \quad (2.1)$$

From equation (2.1), the optimum cycle time that minimizes the total cost C_i is derived as:

$$T_i^* = \sqrt{\frac{2A_i}{h_i r_i (1-\rho_i)}} \quad (2.2)$$

corresponds to the minimum total cost of:

$$C_i^* = \sqrt{2A_i h_i r_i (1-\rho_i)} \quad (2.3)$$

subject to the necessary condition for feasibility of the solution:

$$(\textstyle\sum_i \frac{\sigma_i}{T_i}) \le 1 \quad (2.4)$$

In order to obtain the solutions, four common approaches had been introduced in literatures:

1. Independent Solution

Independent Solution was first introduced by Bomberger (1966). The purpose of introducing this approach was to set a lower bound (LB) to the total cost. Under this approach, basic lot sizing techniques are applied to each item as if it were the only item being produced. The objective function of this problem is to minimize the total cost C stated as:

$$C_i = \frac{s_i}{T_i} + h_i . r_i . (1-\rho_i) . \frac{T_i}{2} \quad (2.5)$$

while the decision variables are the cycle times *T* for all items, subject to the constraints:

a. All values of cycle time $T_i \geq 0$ and integer
b. $\sum_{i=1}^{N} \frac{\sigma_i}{T_i} \leq 1$

2. Common Cycle Approach

Common Cycle approach allows all items to have a single common cycle time. In Common Cycle (CC) approach, a single common cycle is assumed long enough to accommodate the production of each item exactly once in each cycle. Under this CC approach, $T_1 = T_2 = \ldots = T_N = T^*$. Hence, the problem becomes calculating the single common cycle time for all items T^*. This approach was proposed by Hanssmann (1962). The objective function in this problem is to minimize the total cost *C*.

$$C_i = \frac{A_i}{T} + h_i * d_i * (1 - \rho_i) * \frac{T}{2} \qquad (2.6)$$

while the decision variable is the optimum cycle time T^*, subject to the constraints:

a. The value of cycle time $T^* \geq 0$
b. $\sum_{i=1}^{N} \frac{\sigma_i}{T^*} \leq 1$

3. Basic Period Approach

In Basic Period approach, each item is allowed to have different cycle time with restrictions. Under this approach, the different cycle times are allowed to accommodate different cost structures of different items. The cycle time T_i for each item is an integer multiple of a basic period (BP) *W* which is long enough to accommodate the production for all items. This approach was proposed by Bomberger (1966). The objective function in this problem is to minimize the total cost *C*.

$$C_i = \frac{A_i}{n_i W} + h_i * d_i * (1 - \rho_i) * \frac{n_i W}{2} \qquad (2.7)$$

while the decision variables are a set of *n* and a single fundamental cycle time *W*.

This BP approach ensures feasibility of the solution whenever the constraint to the function is satisfied:

$$\sum_{i=1}^{N} \sigma_i = \sum_{i=1}^{n} (s_i + \left(\frac{d_i}{p_i}\right) . n_i . W) \leq W \qquad (2.8)$$

and *n* and *W* are integer.

4. Time-Varying Lot Size Approach

Time-Varying Lot Size approach allows the lot size and cycle time of each product to vary over the time. One of ELSP research under Time-Varying Lot Size approach is by Dobson (1987). Under this approach, the problem can be viewed as deciding on a cycle length *T*, a production sequence f^1, …, f^n, production times t^1, …, t^n, and idle times u^1, …, u^n. These decision variables are chosen so that the schedule is executable, demand is met and total cost is minimized.

2.2. Economic Production Quantity Model with Two Imperfect Modules

The Economic Production Quantity (EPQ) model with two imperfect key modules was proposed by Gong *et al*. (2012). In this model, the production system is dictated by two unreliable key modules (KMs) with their own probability to shift from in-control state to out-of-control state. Key module refers to a certain subsystem of the production system such as the electrical subsystem or mechanical subsystem. These key modules are imperfect; or in other words, may shift from in-control to out-of-control state. When one or two key modules are out-of-control, the production will start to produce defected items.

Under this model, there are four states that may happen in the production system:

a. State O

This state occurs when both KMs are in-control.

b. State A

When the shock comes from the first KM, it becomes out-of-control and is shifted from the state O to the state A. The first KM may shift to state A at a random time with the time-to-shift (X) is an exponentially distributed random variable with mean 1/μ.

c. State B

When the shock comes from the second KM, it becomes out-of-control and is shifted from state O to the state B. The second KM may shift to state B at a random time with the time-to-shift (Y) is an exponentially distributed random variable with mean 1/λ..

d. State AB

State AB happens when the third shock comes from either the first or second KM given that the other KM has been out-of-control.

At the beginning of the production run, both KMs are in-control. By the time the production runs, the shock may occur on either Key Modules. If the shock comes from the first KM, a fixed proportion of α defected items will be produced from the time the production shifts to out-of-control state to the end of the production of a batch. If the shock comes from the second KM, a fixed proportion of β defected items will be produced from the time the production shifts to out-of-control state to the end of the production of a batch. When both KMs are out-of-control, a proportion of (α+β) defected items will be produced.

There are four production uptime (τ) segmentations mentioned by Gong *et al.* (2012) in which the shocks may occur. Those are Ω_1, Ω_2, Ω_3 and Ω_4. In order to calculate the cost incurred by the non-confirming items produced during the out-of-control states, the expected number of nonconforming items as a function of production uptime (τ) should be formulated. Let N(τ) be number of non-conforming items, then the N(τ) for each production uptime segment can be calculated as:

$$N(\tau) = \begin{cases} \alpha(\tau - x) + \beta(\tau - y), & if\ (x,y) \in \Omega_1 = \{0 \le y \le x \le \tau\} \\ \alpha(\tau - x) + \beta(\tau - y), & if\ (x,y) \in \Omega_2 = \{0 \le x \le y \le \tau\} \\ \beta(\tau - y), & if\ (x,y) \in \Omega_3 = \{0 \le y \le \tau \le x\} \\ \alpha(\tau - x), & if\ (x,y) \in \Omega_4 = \{0 \le x \le \tau \le y\} \end{cases}$$

2.3. ELSP Five Key Research Themes

Chan *et al*. (2012) had reviewed ELSP under five key research themes, which are non-uniform production rate; flow shop, multi-machine, or multi-factory; ELSP with return; stochastic problems; and sequence-dependent setup. Following are the discussions of each research theme and their correlations to this research.

a. Non-uniform Production Rate

According to Chan *et al*. (2012), ELSP researches under the non-uniform production rate theme are mostly subject to variable production rate. In basic ELSP, production rate is a deterministic parameter but under the non-uniform production rate theme, it becomes variable parameter.

Variable production rate is occasionally related to the output quality of the production process (Ma *et al*., 2010). Deteriorated production rate affected by imperfect production process is studied by Ben-Daya and Hariga (2000). By employing the Common Cycle (CC) approach in their model, Ben-Daya and

Hariga (2000) released the assumptions that the output of the production facility is always of perfect quality. In their model, the production starts with a perfect quality of output but at a random time may shift to out-of-control stage and start to produce non-conforming items. Taking into account the inspection and restoration process, there are four types of costs included in the model; which are setup cost, holding cost, quality-related cost, and restoration cost. Extending Ben-Daya and Hariga (2000) model, Moon, Giri and Choi (2002) used both the CC-approach and time-varying approach in the analysis of two specific ELSP models, which are imperfect process model (IPM) and imperfect process with inspection and restoration (IPMWIR). The same types of costs are considered in the model but at the end the total cost of those two approaches are compared to show the contrasts.

Ben Daya and Hariga (2000) and Moon, Giri and Choi (2002) explained the ELSP under the imperfect production system. The following notations are used in the model.

i item index, i=1, 2,…, N

d_i demand rate in units per unit time assumed to be deterministic, i=1, 2,…, N

p_i production rate in units per unit time assumed to be constant i=1, 2,…, N

ρ_i $\rho i = \frac{d_i}{p_i}$,i=1, 2,…, N

τ_i $\tau_i = \rho_i T_i$, processing time per lot, i=1, 2,…, N

σ_i $\sigma_i = s_i + \tau_i$, total production time per lot, i=1, 2,…, N

s_i setup time per unit of time per production lot, independent of sequence, i=1, 2,…, N

A_i setup cost per production lot, i=1, 2,…, N

h_i holding cost per unit per unit time, i=1, 2,…, N

u_i constant cost of producing defective product, i=1, 2,…, N

Q_i quality-related cost for item i, i=1, 2,…, N

T_i cycle time for item i, i=1, 2,…, N

t_i length of the production run for item i, i=1, 2,…, N

α_i constant fraction of non-conforming items, i=1, 2,…, N

Based on their models, three ELSP models under the imperfect production system are explained as below:

1. Imperfect Production Model under Independent Solution Approach

In imperfect production system, the total cost is expressed in expected total cost ETC. The formula is written as (Moon, Giri and Choi, 2002):

$$ETC = \sum_{i=1}^{N}[\frac{A_i}{T_i} + (H_i + Q_i)T_i] \quad (2.9)$$

subject to the constraints:

$$\sum_{i=1}^{N} \frac{s_i}{T_i} \leq \kappa \quad (2.10)$$

$$T_i \geq 0, i = 1,2,\dots,N \quad (2.11)$$

where:

$$H = \frac{1}{2}h_i d_i(1-\rho_i),\ Q_i = \frac{u_i \alpha_i \rho_i d_i}{2\theta_i} \text{ and } \kappa = 1 - \sum_{i=1}^{N}\rho_i. \quad (2.12)$$

Corresponding to Karush-Kuhn-Tucker (KKT) necessary condition, the cycle time for each item is defined as:

$$T_i = \sqrt{\frac{A_i+\lambda_1 s_i}{H_i+Q_i-\mu_{1i}}},\ i=1,2,\dots,n \quad (2.13)$$

2. Imperfect Production Model under Common Cycle Approach

Let *t* be the elapsed time of the production stays in in-control state until the shift occurs. In this model, it is assumed that the production stays in out-of-control state producing non-conforming item until the setup of the next production. The expected number of non-conforming item when the system shifts to out-of-control state is given by:

$$E(N_i) = \int_0^{t_i} \alpha_i p_i (t_i - t)\frac{1}{\theta_i}e^{-t/\theta_i}\mathrm{d}t \quad (2.14)$$

After some integration we obtain:

$$E(N_i) = \frac{\alpha_i p_i t_i^2}{2\theta_i} \text{ where } t_i = \frac{d_i T_i}{p_i}. \quad (2.15)$$

The corresponding quality-related cost is also expressed in expected number denoted as *E*(*QC*).

$$E(QC) = \sum_{i=1}^{N}\frac{u_i}{T_i}E(N_i) = \frac{u_i \alpha_i d_i^2 T_i}{2 p_i \theta_i}. \quad (2.16)$$

Thus, combining the expected quality-related cost with the setup cost and inventory holding cost is resulting:

$$E(TC) = \sum_{i=1}^{N}[\frac{A_i}{T_i} + (H_i + Q_i)T_i]. \quad (2.17)$$

In the Common Cycle (CC) approach, a single *T* (cycle time) is employed to all items such that $T_1= T_2=\dots= T_n=T^*$ is minimized by:

$$T^* = \sqrt{\frac{A}{H+Q}} \quad (2.18)$$

where:

$$A = \sum_{i=1}^{N} A_i,\ H = \sum_{i=1}^{N} H_i \text{ and } Q = \sum_{i=1}^{N} Q_i \qquad (2.19)$$

subject to:

$$\sum_{i=1}^{N} (s_i + \frac{d_i T}{p_i}) \leq T. \qquad (2.20)$$

Based on the literatures being reviewed, no papers had been found discussing about ELSP with non-uniform production rate due to imperfection of two key modules. This is the first research gap found from the literatures reviewed.

b. Flow Shop, Multi-machine, or Multi-Factory

This research theme is not very popular in the ELSP focus. This is due to the complexity of ELSP problem even with only single machine or stage or facility. The complexity increases when more machines or facilities are included in the problem. In the field of multi-facility ELSP, Chan *et al*. (2012) combined genetic approach and integer programming in order to solve multi-facility ELSP. Taking into account the setup cost and holding cost, Chan *et al*. (2012) proposed the genetic algorithm for solving Bomberger's problem. The other research on multi-machine ELSP was proposed by Graves (1979). Two machines were considered in this problem with also setup and holding cost taken into consideration. Both papers reviewed are assuming that the items produced are of perfect condition. Besides, this research only considers scheduling of a single machine with two imperfect key modules. These are the second research gap found from the literatures reviewed under flow shop, multi-machine and multi-factory research theme.

c. ELSP with Returns

In respond to the environmental problems, the economic lot scheduling problem was also drawn towards return consideration. This makes the problem more complex since the scheduling is now done for both the new item to be produced and the returned items. Tang and Teunter (2006) focused their research on a specific ELSP problem which is ELSP with return (ELSPR). In order to solve the problem in a car manufacturer as a case being studied, the common cycle (CC) approach was used. The final result showed a cost reduction of 16% after the application of the model. Two types of setup costs are incurred in this model, since the remanufacturing of a certain type of item, even following the manufacturing of the same type done previously, required setup process. Thus, three types of costs are considered in this model, which are the manufacturing and remanufacturing setup cost, and the holding cost. In contrast with Tang and Teunter (2006), the author restricts this research by scheduling only the production of newly-produced items when returned items are excluded.

d. Stochastic Problems

As the basic assumptions in ELSP, the deterministic natures of the demand and production have been released by some researchers to bring the problem closer to the reality. Federgruen and Katalan (1996) included the setup cost, holding cost and the backlog cost in their model, as well as allowing the idle time to be possibly added after a certain production run. They apply a simple but rich class of strategy in the item scheduling which is; the facility will continue processing the item assigned to it until a specific target inventory is reached or the production of a specific batch has been completed. Extending the stochastic ELSP with the lost sales consideration, McKay (1999) worked on single machine scheduling problem in the stochastic demand nature. In his dissertation, McKay started with scheduling single item in single machine, then extended the scope to the ELSP problem by considering both the setup and holding cost in the model. McKay (1999) used the dynamic algorithm in the production sequencing. Since this research does not relax the assumption of deterministic demand and production rate, the fourth research gap is formed.

e. Sequence-Dependent Setup

The other relaxation of ELSP assumption that brought it more realistic is the sequence-dependent setup. Miller *et al*. (1999) used the hybrid genetic algorithm (HGA) to solve the specific case of ELSP with sequence-dependent setup which is SSSDP. The objective of using HGA was to minimize the cost incurred of setup, inventory holding and backlog (Miller *et al.*,1999). Analyzing on only single machine/facility, the research took place at a manufacturing of automotive parts. In contrast with Miller *et al*. (1999), this research lies on the basic ELSP assumption of sequence-independent setup.

f. Other Themes

A lot more papers fell outside Chan *et al*. (2012) five key research themes. Focusing on the feasibility test of economic lot scheduling problem, Hsu (1983) developed general feasibility test of scheduling lot size for several products on one machine by considering setup cost and holding cost. Two ELSP basic approaches were used, which are the common cycle and basic period. Dobson (1987) used another basic ELSP approach which is time-varying lot size in order to solve Bomberger's problem. Dobson (1987) considered the setup and holding cost into the model. Elhafsi and Bai (1997) relaxed one of ELSP assumptions by allowing backorder. Since the backorder is allowed, therefore there is one additional type of cost added to common ELSP problem which is backlog cost.

In extension to what Dobson (1987) has done, Bae *et al*. (2014) worked on modified Kuhn and Liske's and Mallya's problem. By using time-varying approach, Bae *et al*. (2014) took into account the setup cost and holding cost. Karalli and Flowers (2006) worked on special ELSP theme which is multiple-family by adding safety stock into consideration. Taking place in a manufacturer of quartz tubing for lighting applications, Karalli and Flowers (2006) considered two types of setup costs, which are minor and major setup cost, as well as the holding cost. Minor setup cost incurred when setup is needed for another batch within one family. Major setup cost incurred when setup is needed for another family. Focusing only on one single facility, Karalli and Flowers (2006) used the basic period approach to solve the problem.

Genetic algorithm was also widely used as the heuristic approach to ELSP by some researchers. The pure genetic search approach was introduced by Chatfield (2007). By using Bomberger's problem as the data, Chatfield (2007) developed the GA and compared the result in term of total cost with another approaches. Moon, Silver and Choi (2002) used the hybrid genetic algorithm to solve Mallya's problem on ELSP. Moon, Silver and Choi (2002) used the same lower bound scheme as it was used in Dobson heuristic to find the x_is, therefore they called it *hybrid*.

The other algorithm was proposed by Raza *et al*. (2006), which was the tabu search (TS) algorithm for solving economic lot scheduling problem. Raza *et al*. (2006) was using Bomberger and Mallya's problem in the analysis to show the improvement that the Tabu Search Algorithm had compared to the other previous algorithm proposed. Raza *et al*. (2006) covered two types of costs in ELSP which are setup cost and inventory holding cost.

2.4. Gap Analysis and Research Contributions

After reviewing 16 papers on ELSP, these papers are classified based on Chan *et al*. (2012) five key research themes in ELSP, basic ELSP assumptions and the approaches used. Papers are categorized under non-uniform production rate; flowshop, multi-machine or multi-factory; ELSP with returns; stochastic problems; sequence-dependent setup; and the basic assumptions as contrast comparisons are uniform production rate; deterministic problems; sequence-independent setup; multi-family ELSP; ELSP with backorders and the four common approaches used in previous papers. As it can be seen in Table 2.1. below, no paper was found focusing on ELSP in imperfect production system with two key modules (KMs). This is the research gap with previous papers on ELSP. This research contributes to the management science, especially the Economic Lot Scheduling Problem, by relaxing the assumption of perfect production process. The relaxation of this assumption is taken through the possibility that the system may shift from in-control to out-of-control condition due to certain shock coming from either Key Modules. Adding up to the two common costs included in the ELSP which are setup cost and inventory holding cost, quality-related cost of producing defected item is employed in this research.

Using modified data of Bomberger's stamping problem, this research presents the mathematical model on spreadsheet such that the calculation of the cycle time *T* can be done automatically through the use of solver function for ELSP in perfect production system and imperfect production system with one key module. The development of the models on spreadsheet is started from perfect model under IS, CC and BP approach, imperfect model with one key module under IS and CC approach, then followed by imperfect model with two key modules under IS and CC approach. The spreadsheet models on perfect production system and imperfect production system with one key module will be the base model to develop the spreadsheet model of ELSP in imperfect production system with two key modules.

Table 2.1. ELSP Papers Classified by Chan's Key Themes and ELSP Basic Assumptions

	Number of Key Module (KM)		Flow Shop, Multi-Machine or Multi-Factory	ELSP with Returns	Stochastic Problems	Deterministic Problems	Setup		Production Rate		Multiple Family ELSP	Backorder	Approach Used			
	1 KM	2 KM					Sequence-Dependent	Sequence-Independent	Uniform	Non-uniform			IS	CC	BP	TV
Ben-Daya and Hariga(2000)	√					√		√		√			√	√		
Moon, Giri and Choi (2002)	√					√		√		√			√	√		√
Chan *et al*.(2012)			√			√		√	√				√			
Graves (1979)			√			√		√	√				not specified			
Tang and Teunter (2006)				√		√		√	√					√		
Federgruen and Katalan (1996)					√			√	√				not specified			
McKay (1999)					√			√	√				not specified			
Miller *et al*. (1999)						√	√		√				not specified			
Elhafsi and Bai (1997)						√		√	√			√		√		
Bae *et al.*(2013)						√		√	√							√
Hsu (1983)						√		√	√					√	√	
Dobson (1987)						√		√	√							√
Karalli and Flowers (2006)						√		√	√		√				√	
Moon, Silver and Choi (2002)						√		√	√							√
Chatfield (2007)						√		√	√				genetic approach			
Raza *et al*. (2006)						√		√	√				tabu search			
Adhisatya (2015)		√				√		√	√				√	√		